Worksheet 10: Sampling distributions

Example 0.84 (Brown eggs)**.** A random sample is a collection of iid random variables: $X_1, \ldots, X_{12} \sim N(65, 2^2)$ (weights of 12 eggs to be selected).

Example 0.85. Suppose you toss a coin (with probability of heads p) independently for n times, and let $X_1, \ldots, X_n$ denote the numerical outcomes of single trials: 1 (heads) or 0 (tails). This constitutes a random sample from the Bernoulli(p) distribution because

$$X_1, \ldots, X_n \overset{\text{iid}}{\sim} \text{Bernoulli}(p).$$

Example 0.86. Let $X_1, \ldots, X_n$ represent n repeated measurements of an object's length/weight. They can be thought of as a random sample from a normal distribution

$$X_1, \ldots, X_n \overset{\text{iid}}{\sim} N(\mu, \sigma^2)$$

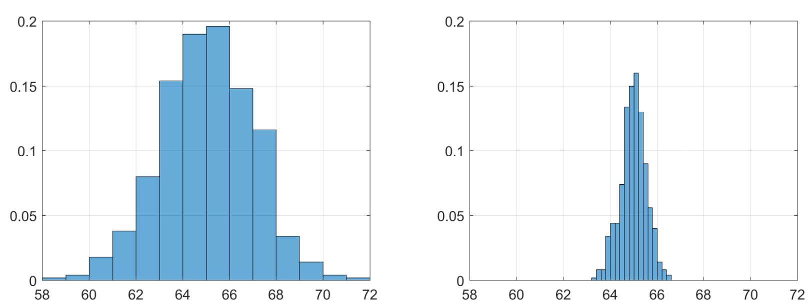
where

- μ: true length/weight (if the measurement process is unbiased)
- σ^2: variance of the measurement error.

Example 0.87. Suppose you *actually* buy a carton of $n = 12$ eggs from the farm and measure their weights individually. Then you may obtain a data set like the following (called a **specific sample**):

$$x_1 = 65.4,\ x_2 = 65.0,\ x_3 = 64.8,\ x_4 = 65.1,\ x_5 = 64.8,\ x_6 = 64.4,$$
$$x_7 = 65.0,\ x_8 = 65.1,\ x_9 = 65.5,\ x_{10} = 64.8,\ x_{11} = 64.8,\ x_{12} = 65.2$$

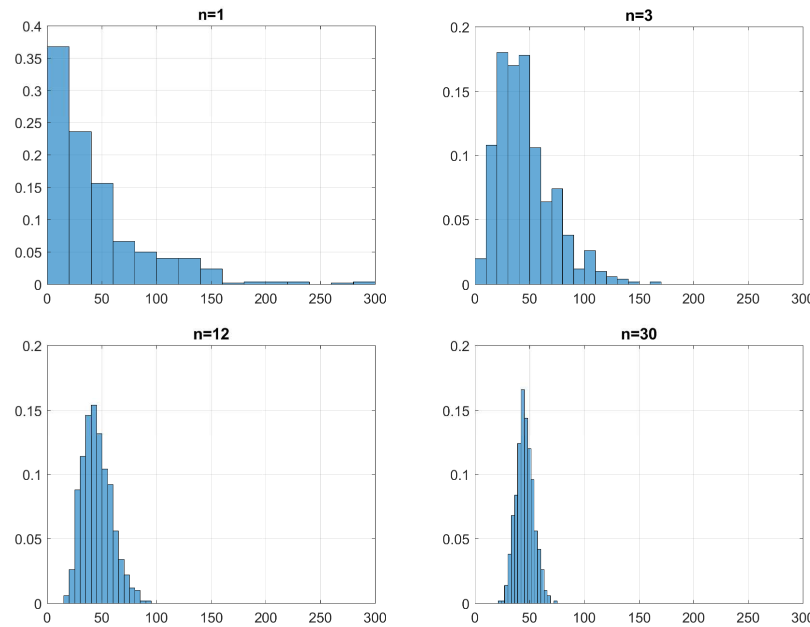
Example 0.88. Weights of 500 single eggs (left) and average weights of 500 cartons (right), all selected at random.



Example 0.89. In the egg weight example, suppose the population distribution is $N(65, 2^2)$. For a sample of size 12, what is the probability that $\bar{X}$ is within 65 ± 1? What about an individual egg? (Answers: .9167, .3829)

Example 0.90. In the library elevator of a large university, there is a sign indicating a 16-person limit as well as a weight limit of 2500 lbs. When the elevator is full, we can think of the 16 people in the elevator as a random sample of people on campus. Suppose that the weight of students, faculty, and staff is normally distributed with a mean weight of 150 lbs and a standard deviation of 27 lbs. What is the probability that the total weight of a random sample of 16 people in the elevator will exceed the weight limit? (*Answer*: .1762)

Example 0.91. Suppose salaries of all SJSU employees follow an exponential distribution with average salary = 45K (which means that $\lambda = \frac{1}{45}$). We draw a random sample of size n from the population, and compute the sample mean $\bar{X}$. We display the histograms of the simulated values of $\bar{X}$ through 500 repetitions for each of $n = 1, 3, 12, 30$.



Example 0.92 (cont'd)**.** Find $P(\bar{X} > 55)$ when $n = 30$. *Answer: 0.1118 (CLT), 0.1157 (exact)*