Worksheet 6: Discrete random variables

Example 0.56. Bernoulli examples:

- (Toss a fair coin) $X = 1$ (heads) and 0 (tails).
- (Randomly select a ball from an urn that has 10 red and 20 blue balls) Let $Y = 1$ (if the selected ball is red) and 0 (otherwise).
- (Randomly select an individual from a population 40% of which have certain characteristic) Let $Z = 1$ (if the selected individual has the characteristic) and 0 (otherwise).

Example 0.57. Binomial examples:

- (Toss a fair coin 10 times) $X = \#$heads
- (Answer 10 multiple-choiced questions by random guessing) $X=$ #correctly answered questions
- (Draw with replacement 10 balls at random from an urn containing 30 red and 20 blue balls) $X=$ #red balls selected

Example 0.58. What is the probability of getting 0 heads in 10 independent flips of a fair coin? Exactly 1 head? At least two heads?

Example 0.59 (Answer 10 multiple-choiced questions by random guessing)**.** Let $X = \#$correctly answered questions. Find $P(X = x)$ for $x = 0, 2, 9$.

Example 0.60. Consider the experiment of drawing, without replacement, n voters at random from the whole pool of N that are registered, r of which support certain presidential candidate. Let $X = \#$supporters of the candidate in the selection. Then $X \sim \text{HyperGeom}(N, r, n)$. In reality, both r and N are typically large (e.g., in the order of millions) and n is only around a thousand. Accordingly, we have that $X \overset{\text{approx}}{\sim} B(n, p = \frac{r}{N})$.

Example 0.61. The following random variables have geometric distributions:

- (Repeatedly flip a coin until the first head appears) X = total number of flips needed.
- (Repeatedly draw balls with replacement from an urn containing 5 red balls and 5 blue balls) X = #selections required to obtain a red ball for the first time.

Example 0.62. Suppose X has a geometric distribution with $p = \frac{1}{2}$. Find $P(X = 4)$ and $F(X \geq 4)$.

Example 0.63. Suppose, on average, 2.2 hurricanes hit a region each year. Let X = #hurricanes next year. Find the following probabilities:

- $P(X = 0) =$
- $P(X = 1) =$
- $P(X = 2) =$
- $P(X \geq 2) =$

Example 0.64. Verify that for $X \sim \text{Pois}(2.2)$, $P(X = 2) = 0.2681$. In contrast, if $X \sim B(n = 365, p = \frac{2.2}{365} = 0.0060)$, then $P(X = 2) = 0.2689$.

Example 0.65. The first draft of a probability textbook has 600 pages. Assume that the probability of any given page containing at least one typographical error is 0.015 and the numbers of errors on all the pages are mutually independent. Let T be the total number of pages which have at least one typographical error. Find the probability that $T = 9$. *Answer: .1328 (exact), or .1318 (approx)*