CAMCOS Verizon Project

Adaptive Spectral Clustering for High-Dimensional Sparse Count Data

with Applications to Document and Web User Clustering

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Team Members: Shiou-Shiou Deng, Sonia Kong, Nate Kotila, Rachel Li, Ryan Quigley, Andrew Zastovnik

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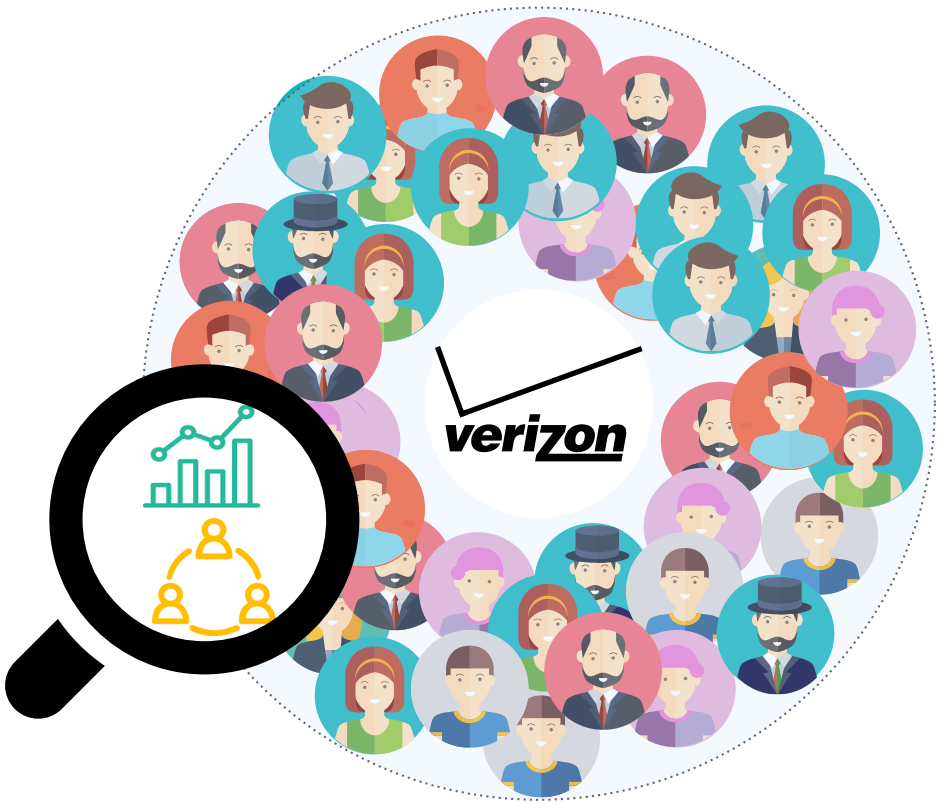
Outline

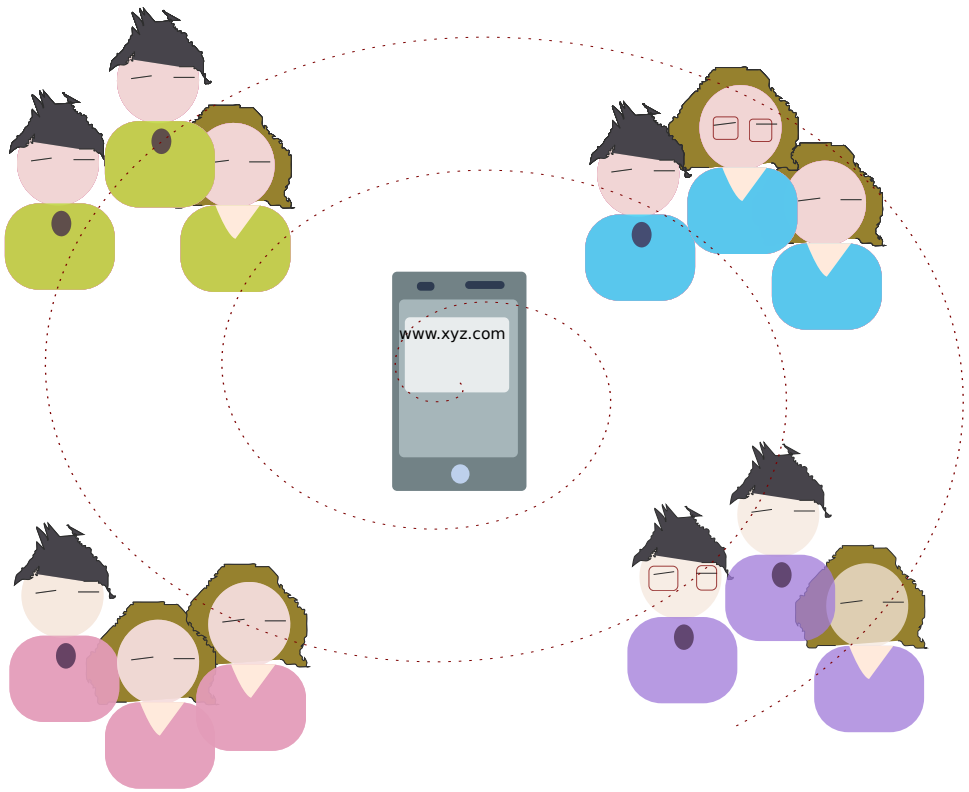
Project Overview
20 Newsgroup Dataset
Data Processing
Dimensionality Reduction
Similarity
Spectral Clustering
Insights

Project Overview - Fengmei

From users' demographic and browsing history data, get insights about customer segmentation, preference prediction, time drift...



Our focus: Discover audience with similar characteristics



Verizon Simulated Data

User	tldAggScore
Id 1	{web1: 1, webm:1}
Id 2	{webm:1}
Id 3	None
...	...
Id n	{web k:1}

999, 937 users

Filter out missing

Verizon Data

(web visits part)

	Web 1	2	...	m
Id 1	1	0	...	1
Id 2	0	0	...	1
...	...	...	...	...
Id n	0	0	...	0

Dense Matrix

User	Web	Frequency
Id 1	Web 1	1
Id 1	Web m	1
Id 2	Web m	1
...	...	...
Id n	Web k	1

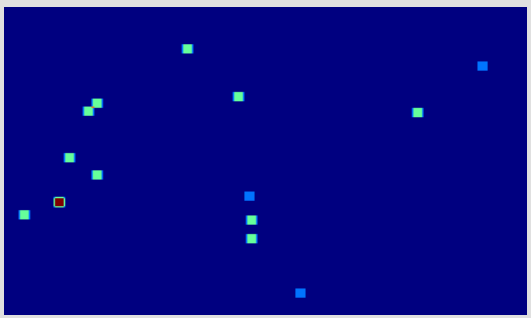
Sparse Matrix

High-Dimension

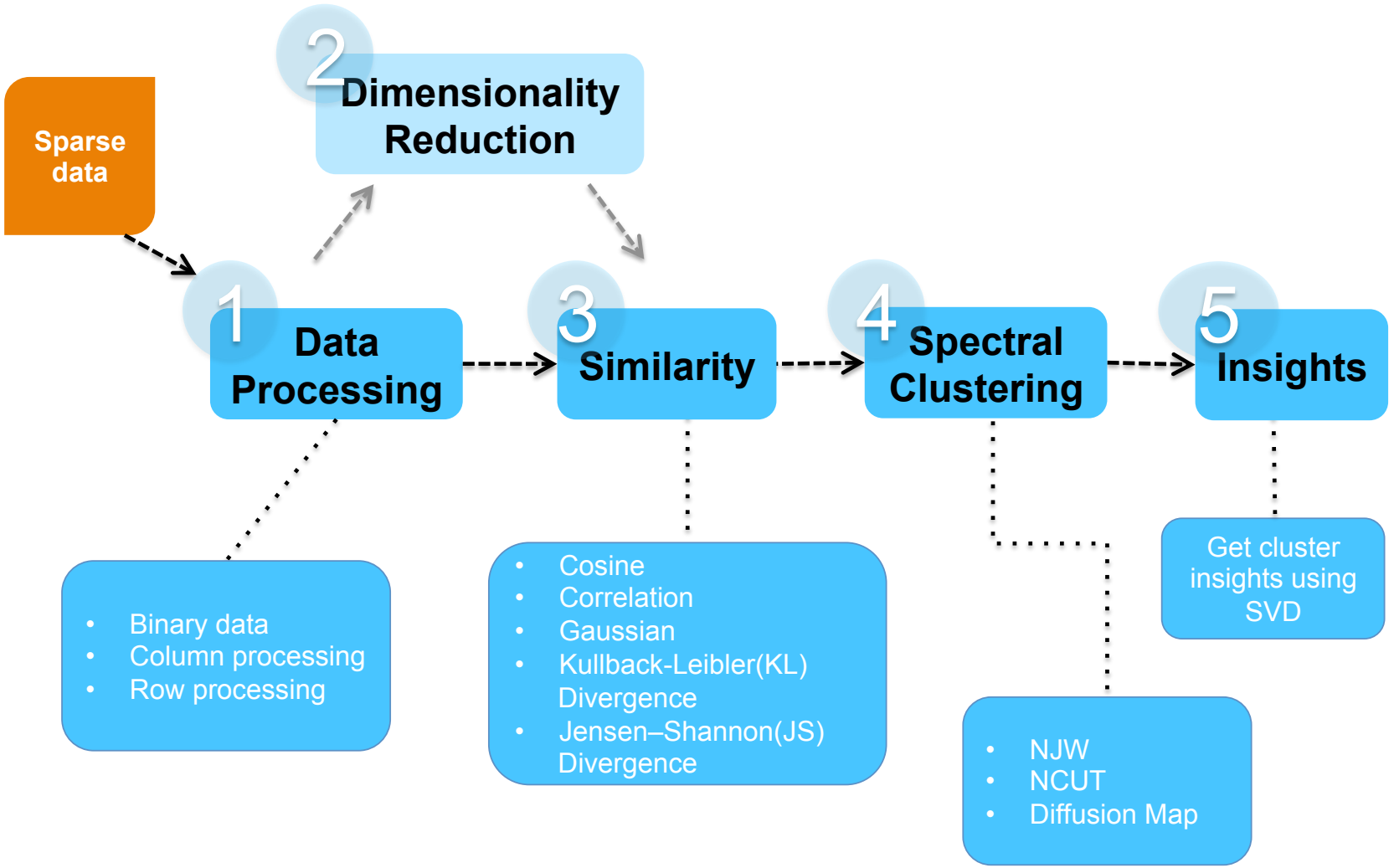
$$330K * 175K$$

High Sparsity

$$\frac{591K}{330K * 175K} = 0.001\%$$



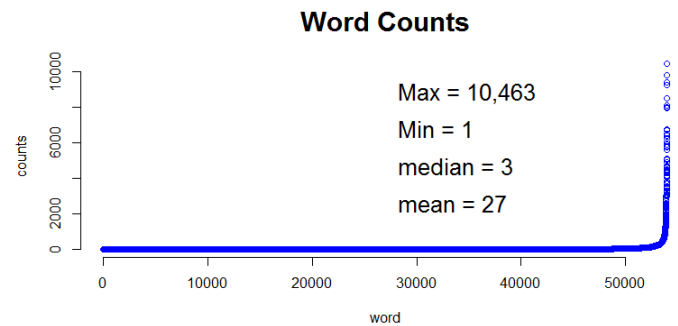
Process Framework

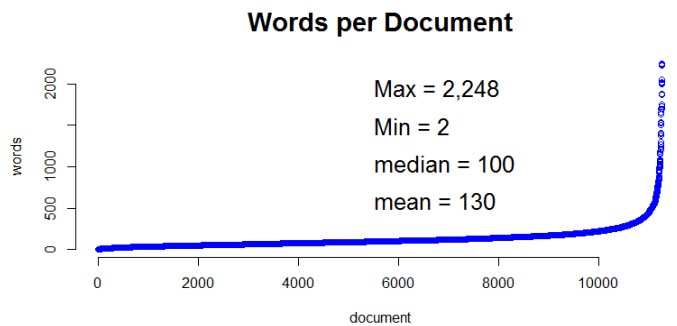


20newsgroup dataset - Rachel

20newsgroup dataset

- **Similar to Verizon dataset (proof of concept for the project):**
  - count documents – high dimension – high sparsity
- **Basic information**
  - newsgroup documents on 20 topics
  - ground truth available
  - row(documents): 11,269
  - column (words): 53,975
  - Density: 0.24%





20newsgroup dataset

Original Sparse Form

docID	wordID	count
doc 1	word 1	1
doc 1	word 2	4
doc 1	word 3	3
doc 1	word 4	1
doc 2	word 1	9
doc 2	word 2	1
doc 2	word 4	2
doc 3	word 1	2
doc 3	word 5	3
...	...	...
doc m	word n	...

Dense Matrix Form

	word 1	word 2	word 3	word 4	word 5	...	word n
doc 1	1	4	3	1	0	...	...
doc 2	9	1	0	2	0	...	...
doc 3	2	0	0	0	3	...	...
...	...	...	...	...	...	...	...
doc m	...	...	...	...	...	...	...

Class of the 20newsgroup data

comp.graphics
comp.os.ms-windows.misc
comp.sys.ibm.pc.hardware
comp.sys.mac.hardware
comp.windows.x

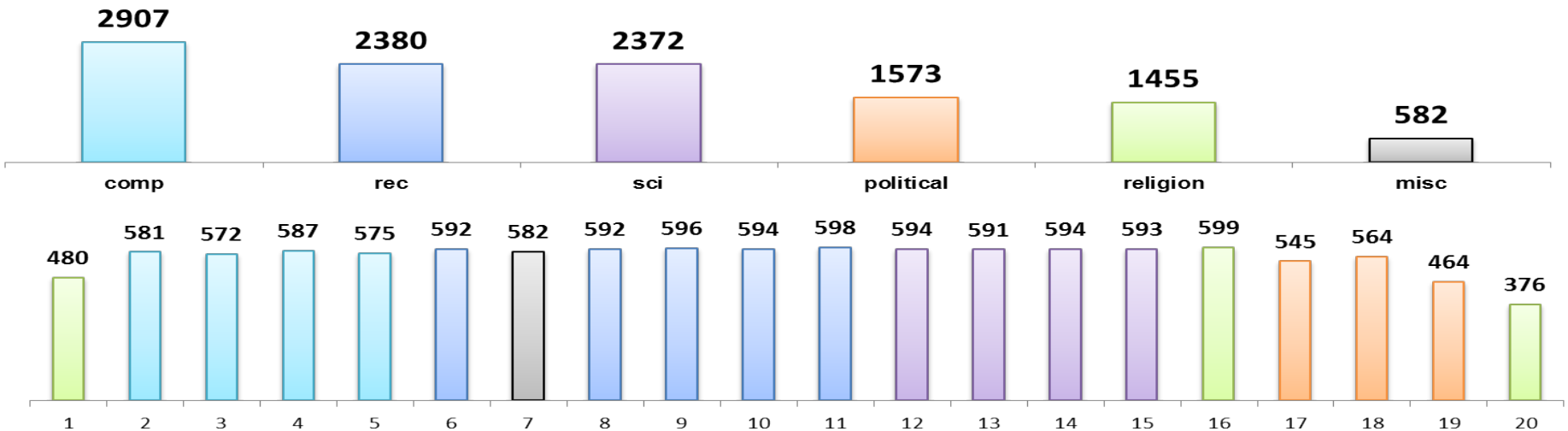
talk.**politics**.misc
talk.**politics**.guns
talk.**politics**.mideast

sci.crypt
sci.electronics
sci.med
sci.space

rec.autos
rec.motorcycles
rec.sport.baseball
rec.sport.hockey

talk.**religion**.misc
alt.**atheism**
soc.**religion**.christian

misc.forsale



Data Processing - Ryan

Data processing steps:

1. Binarization: 0-1
2. Column Processing: trimming and weighting
3. Row Processing: trimming and normalization

Binarization

- Convert all non-zero entries to 1
- Indicator of word occurrence in a document
- Trade-off between loss of information and de-emphasis of high frequency terms

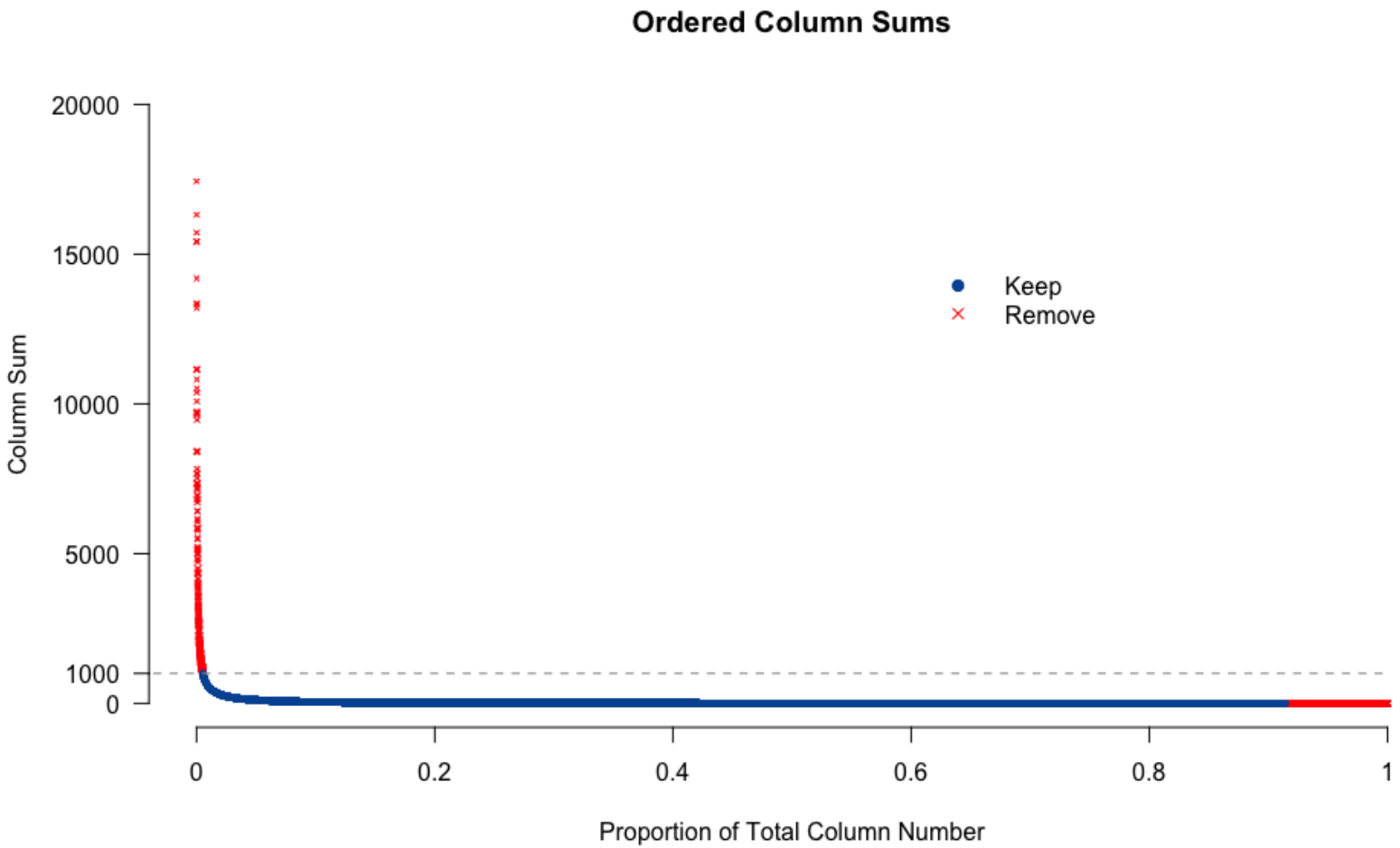
Column Processing

Trimming thresholds:

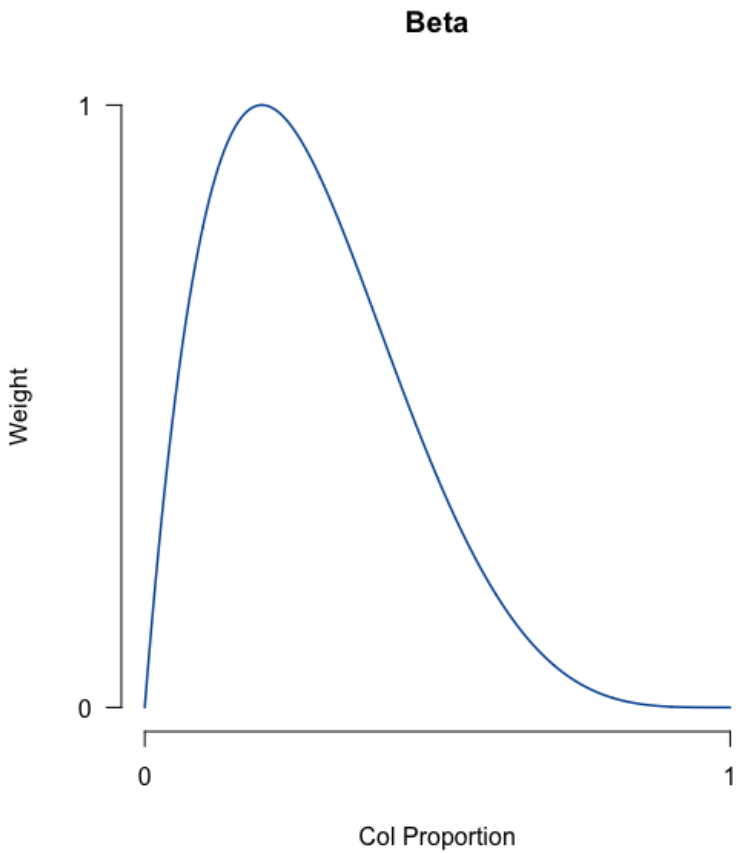
- Min document occurrence: 1
  - Removes 9%
- Max document occurrence: >1000
  - Removes 0.5%

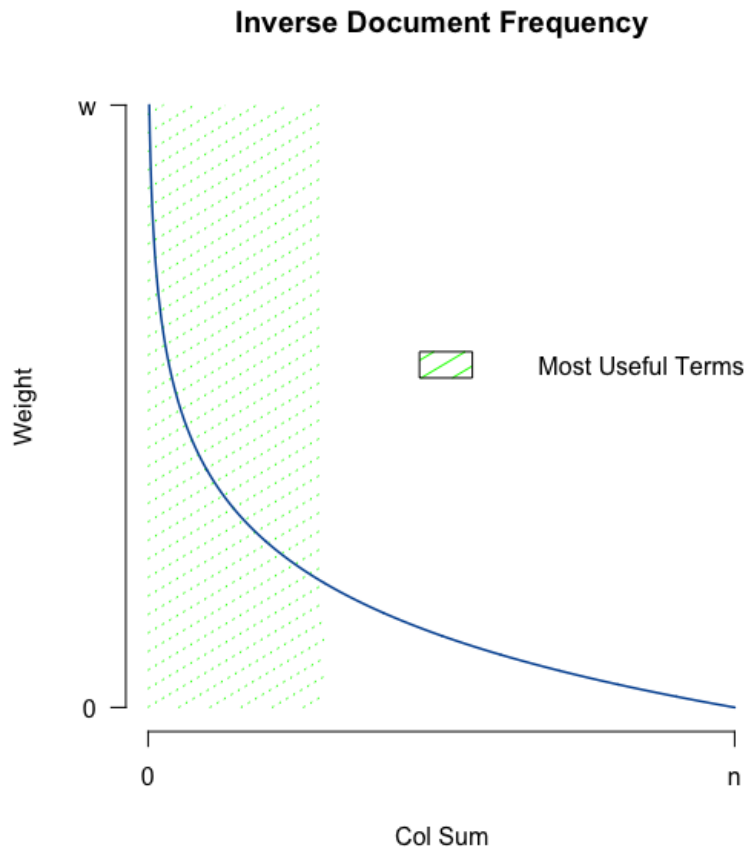
	$Term_1$	$Term_2$	$Term_3$	$Term_4$	. . .	$Term_p$
Doc_1	1	0	0	0	. . .	1
Doc_1	1	1	1	0	. . .	0
Doc_1	0	1	0	0	. . .	1
.						
.					Document Term Matrix	
.						
Doc_N	0	1	1	1	. . .	0
Column Sums	42	1251	7	1		23

20newsgroup dataset



Column Weighting

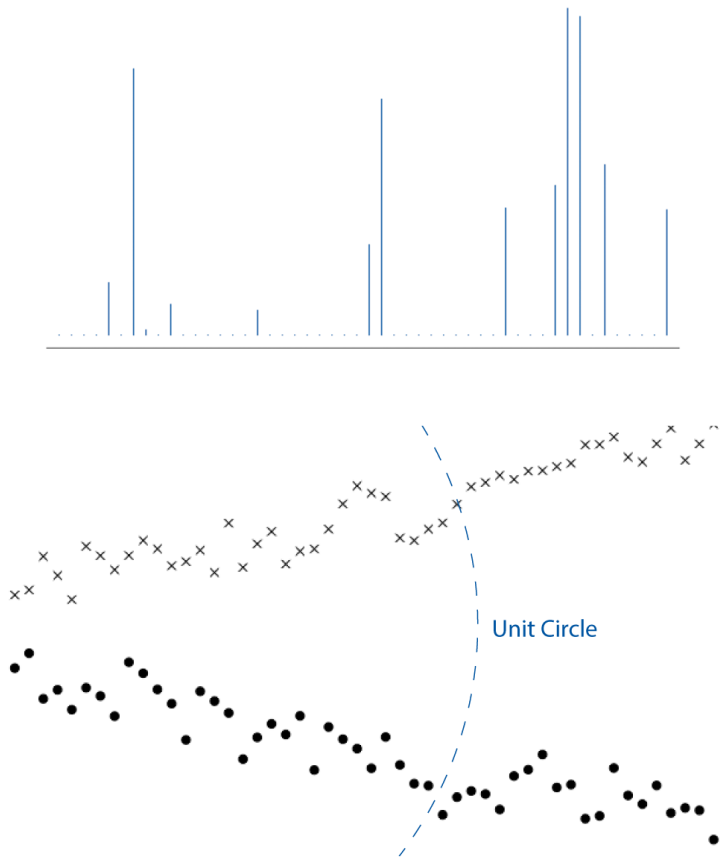




Row Processing

Normalization

- L1 - $\frac{W}{||W||_1}$
- L2 - $\frac{W}{||W||_2}$



Dimensionality Reduction - Qingbin

Motivation

- Feature Extraction
- Curse of dimensionality
  - sparsity, problematic for any method that requires statistical significance
  - Noise, covers the true structure or pattern in the data
  - Time Consuming, Memory demanding

Idea of Latent Semantic Indexing

- Assume lower dimensions reveal latent features of original data space
- Map documents (and terms) to this lower dimensional space
- Reduced data reflects true structure of original data
- Compute document similarity based on Reduced data

SVD (Mathematical LSI)

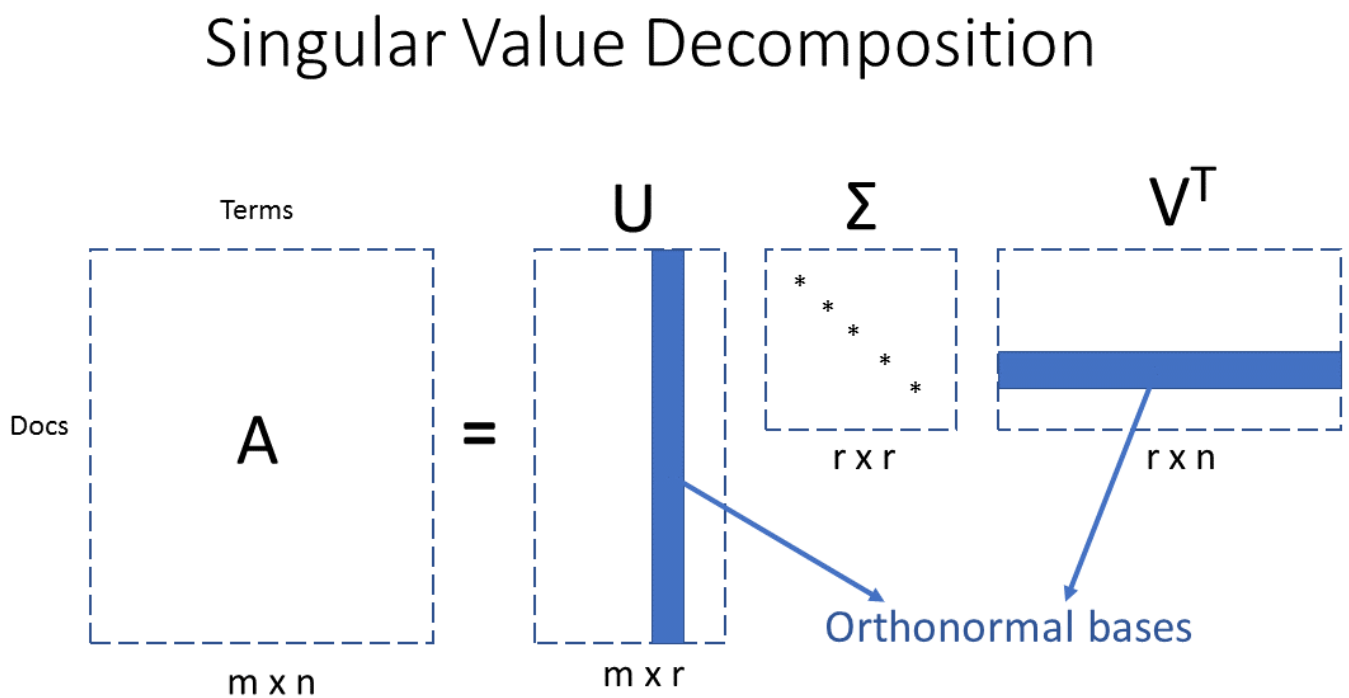
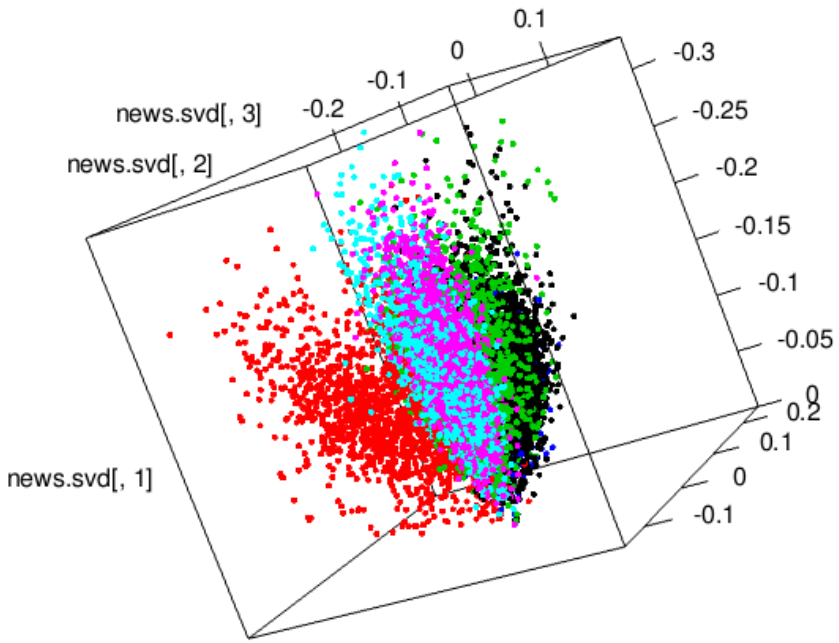


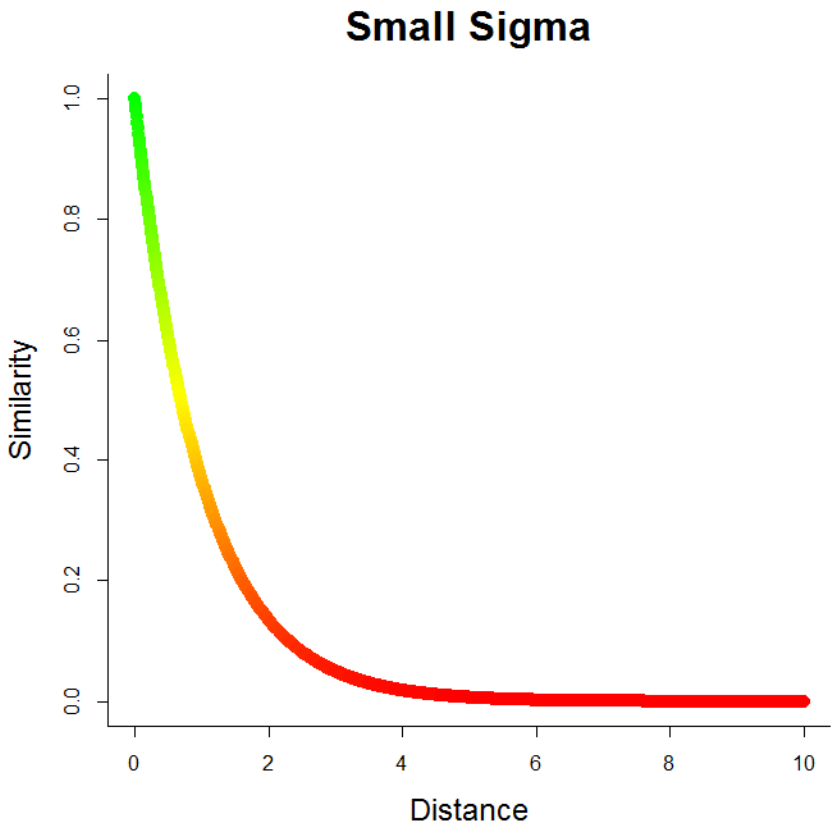
Illustration of 20newsgroup data with SVD

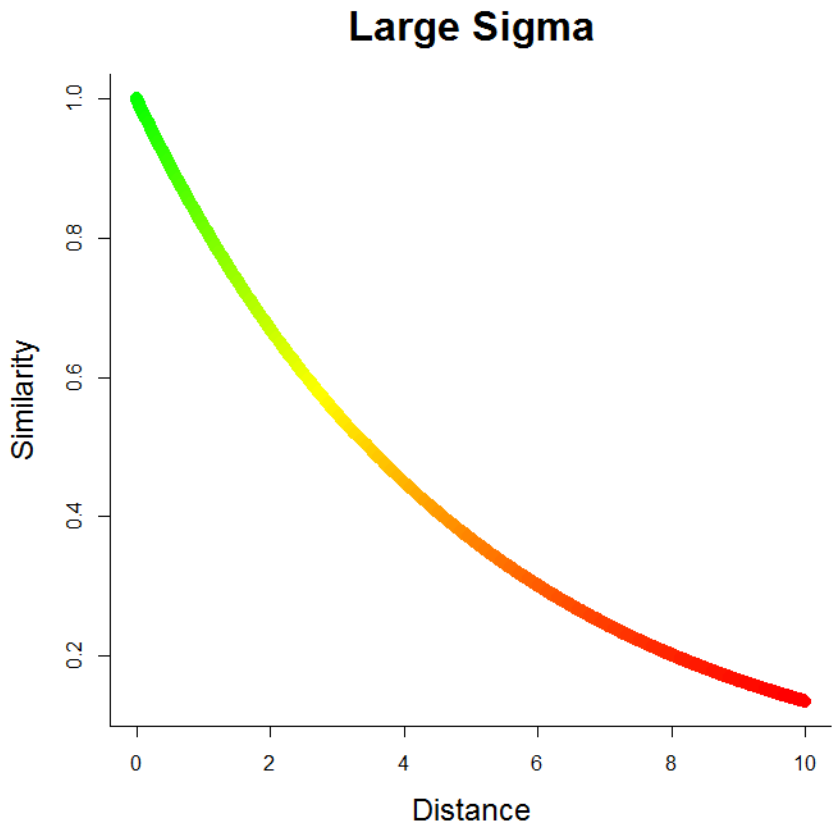
- Significant structure
- Basis for similarity calculation



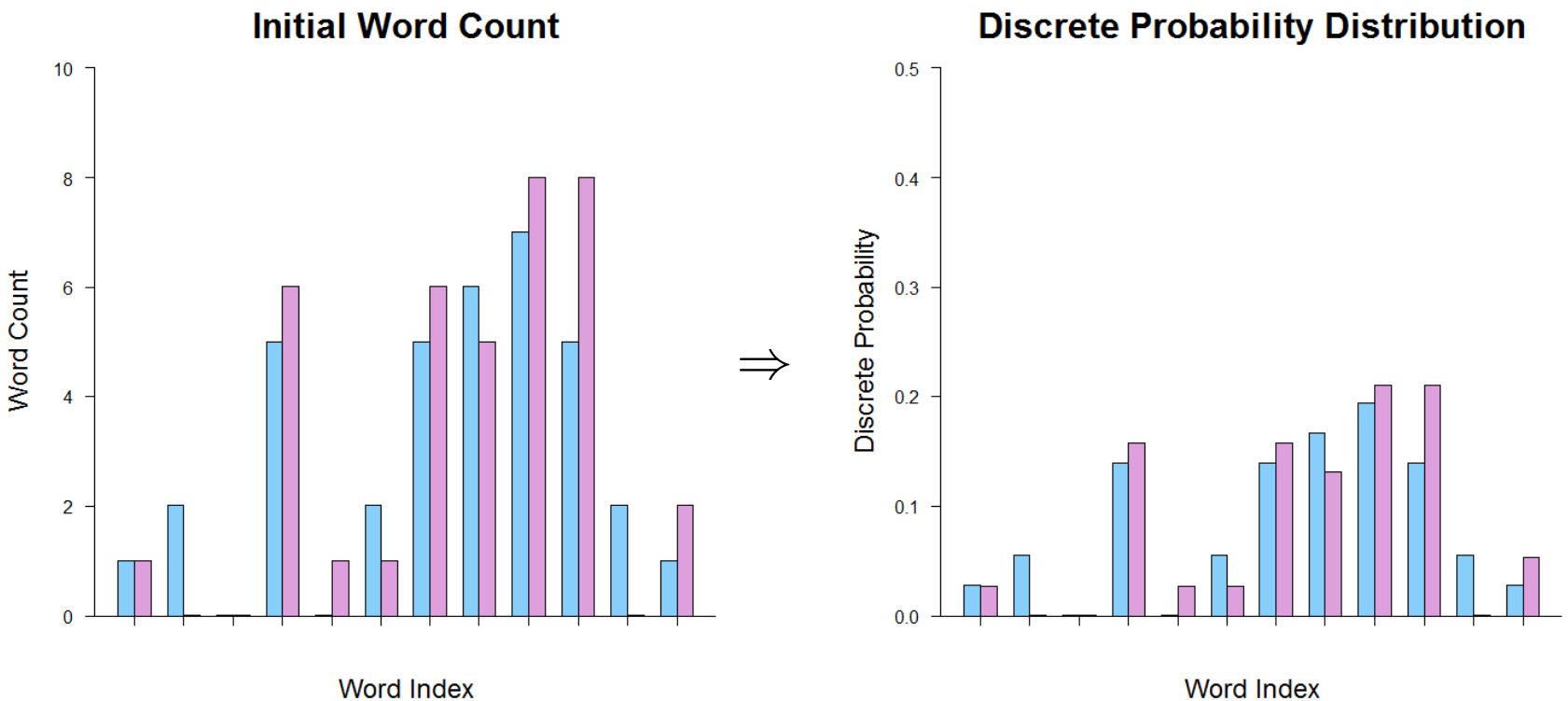
Similarity - Joey

1) Gaussian Kernel: $\text{Sim}(x, y) = e^{\frac{-\text{dist}(x,y)^2}{2\sigma^2}}$





1.1) Kullback-Leibler Divergence:



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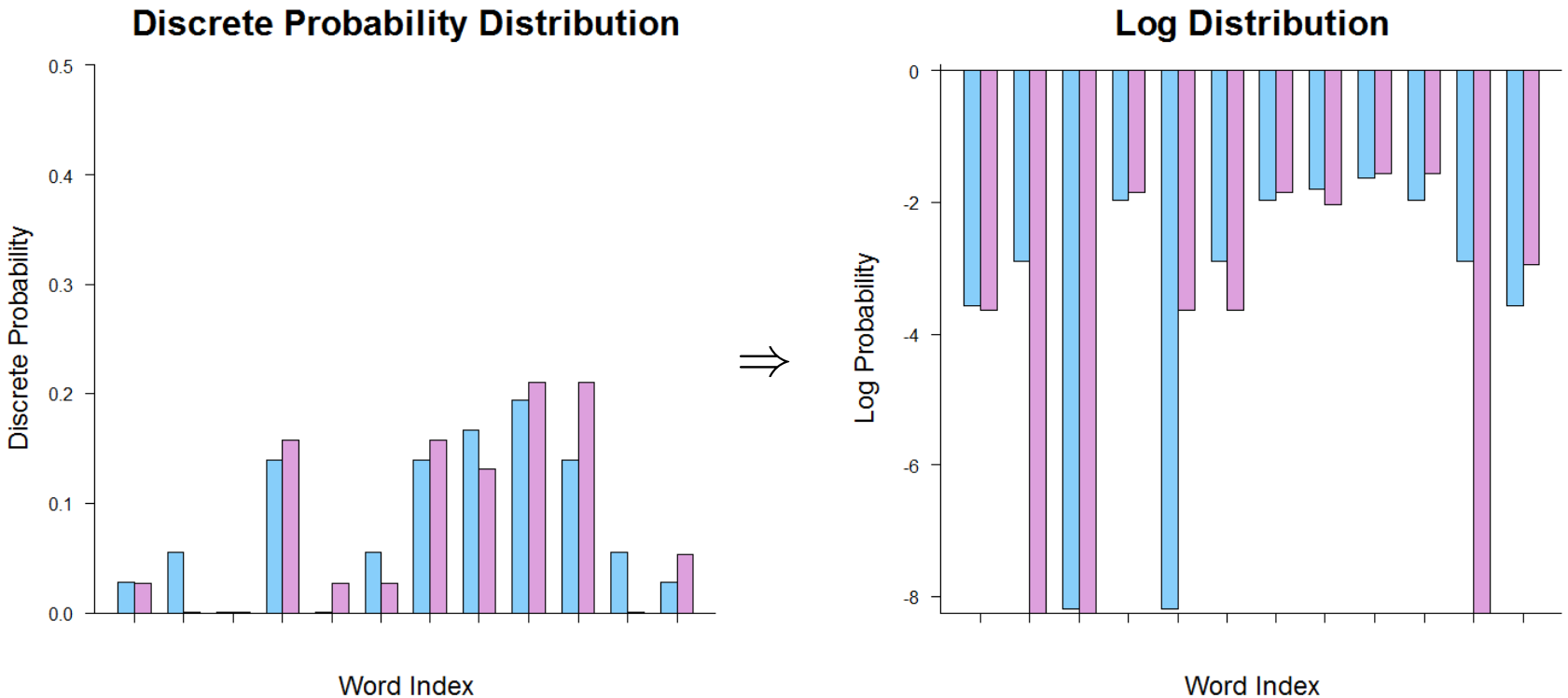

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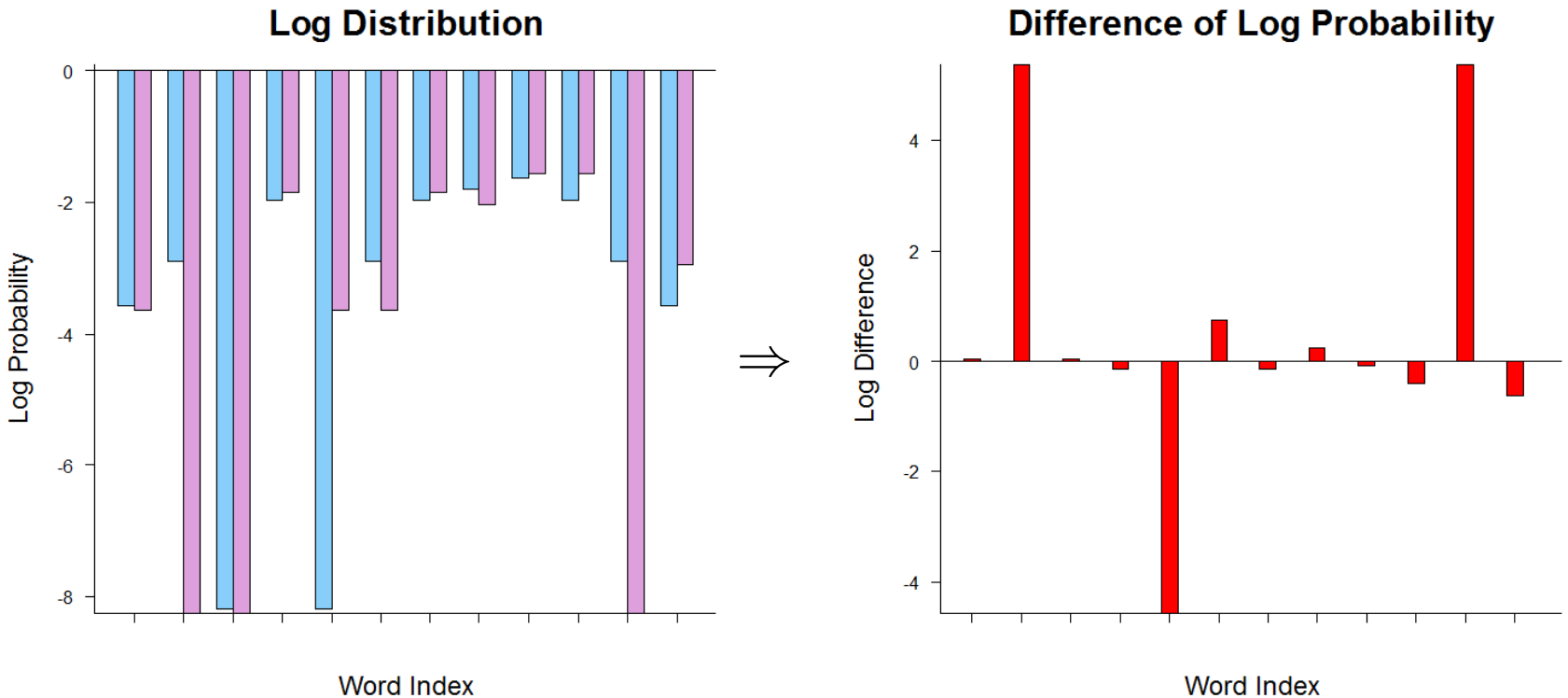

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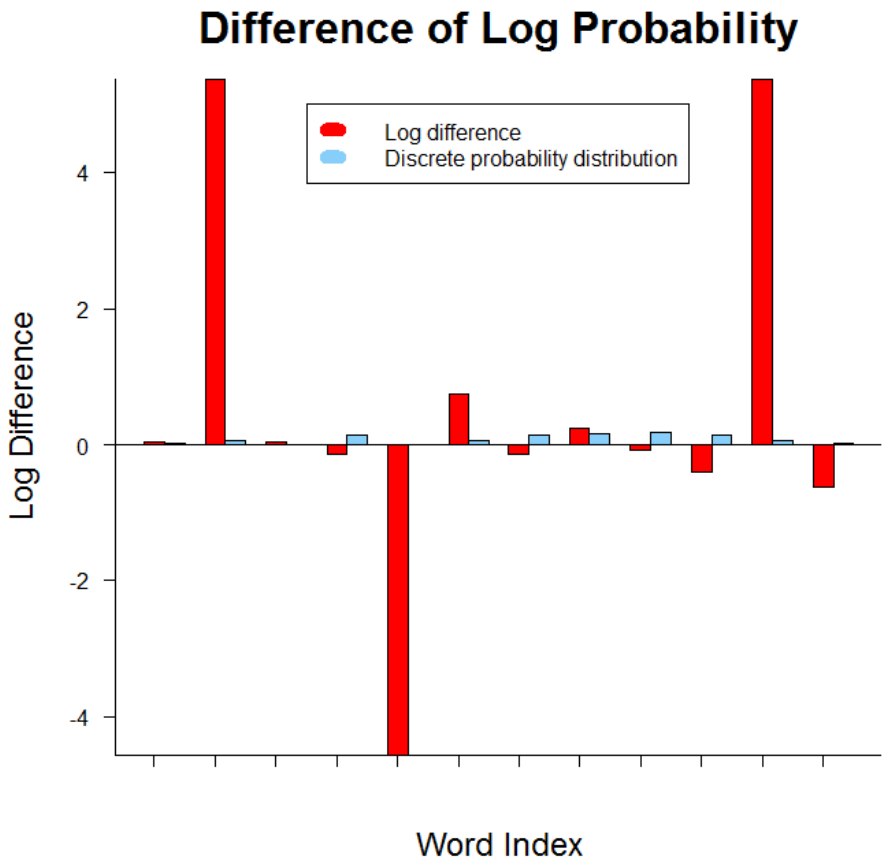


For discrete distributions x and y:

Divergence(x, y):

$$= \mathrm{E}\left[\log(x) - \log(y)\right]$$

$$= \sum_k x_k \left[\log\left(\frac{x_k}{y_k}\right)\right]$$

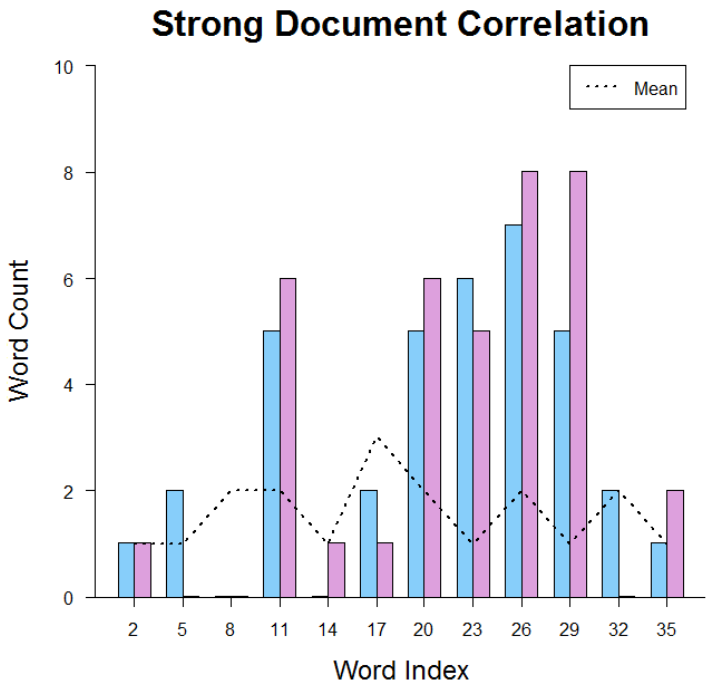
1.2) Jensen-Shannon Divergence:

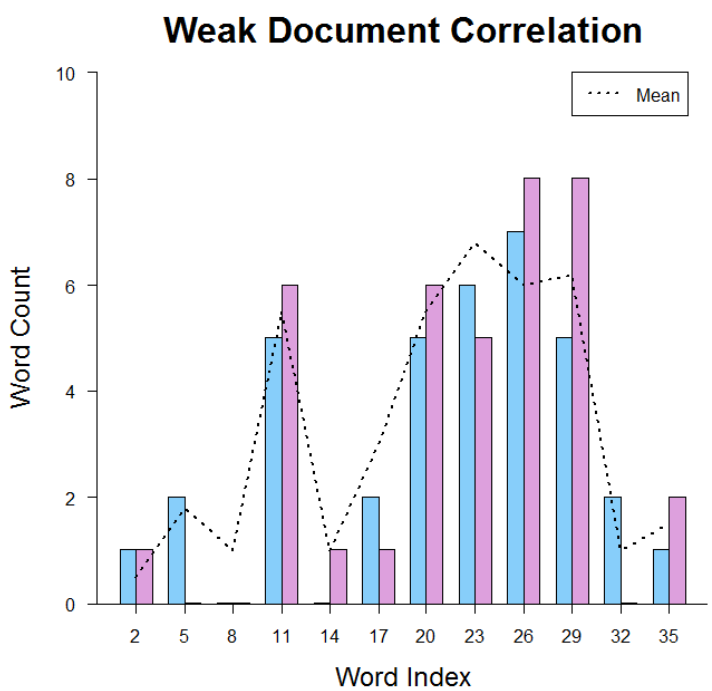
"Average Distribution" Symmetry: $M = \frac{x+y}{2}$

$$\text{JS.Div}(x, y) = \frac{\text{Divergence}(x,M) + \text{Divergence}(y,M)}{2}$$

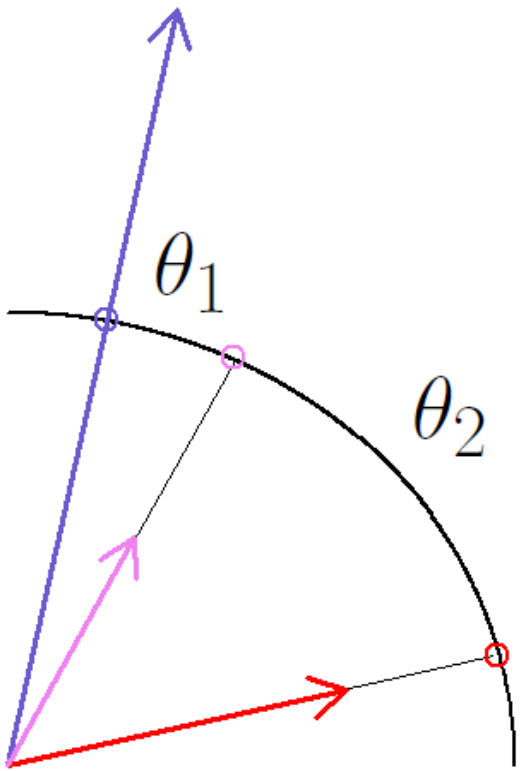
$$\Rightarrow \text{Sim}(x, y) = e^{\frac{-\text{JS.Div}(x,y)^2}{2\sigma^2}}$$

2) Correlation: $\text{Sim}(x,y) = \dfrac{(\vec{x}-\vec{\mu})\cdot(\vec{y}-\vec{\mu})}{\sqrt{||\vec{x}-\vec{\mu}||^2\ ||\vec{y}-\vec{\mu}||^2}}$





3) Cosine: $\text{Sim}(x, y) = \dfrac{\vec{x} \cdot \vec{y}}{\sqrt{||\vec{x}||^2 \, ||\vec{y}||^2}} = \cos(\theta_{xy})$



Outliers Removal - Nate

Outlier Removal

What are outliers?

- Documents that have low connectivity to other documents
- Documents that have low information
  - Very common words
  - Very few words

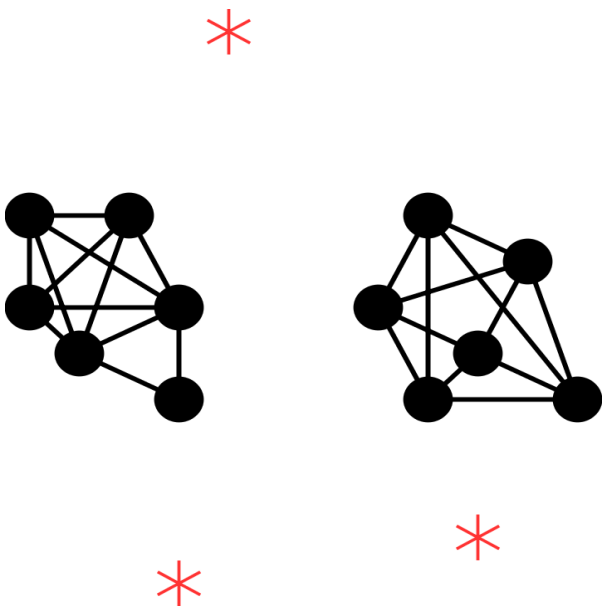
Determining Connectivity or Information

- Row sums of our data matrix:

    1. Raw data tells us how many words are in the document

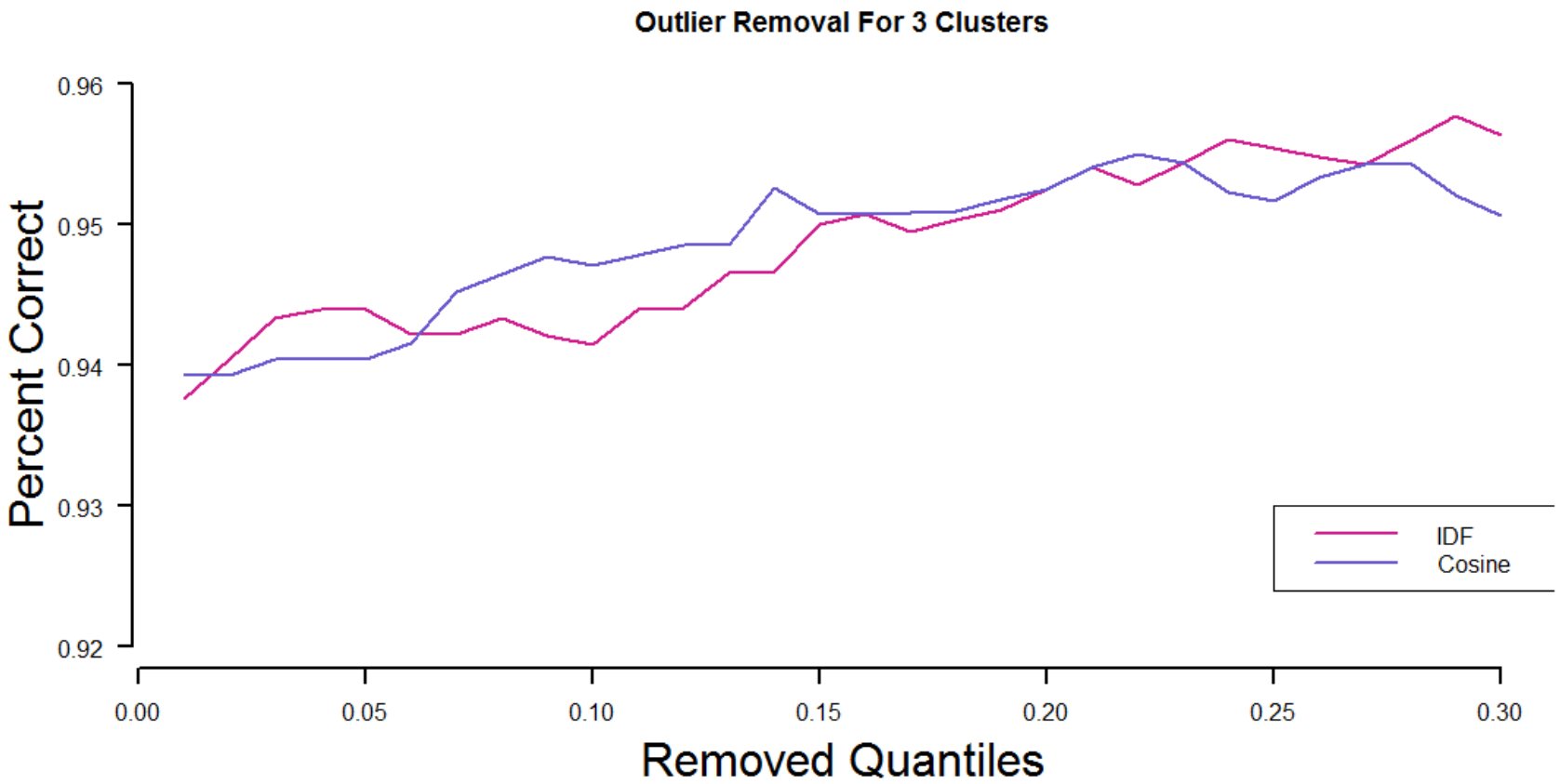
    2. IDF weighted data tells us how many *useful* words are in the document

Outlier Removal

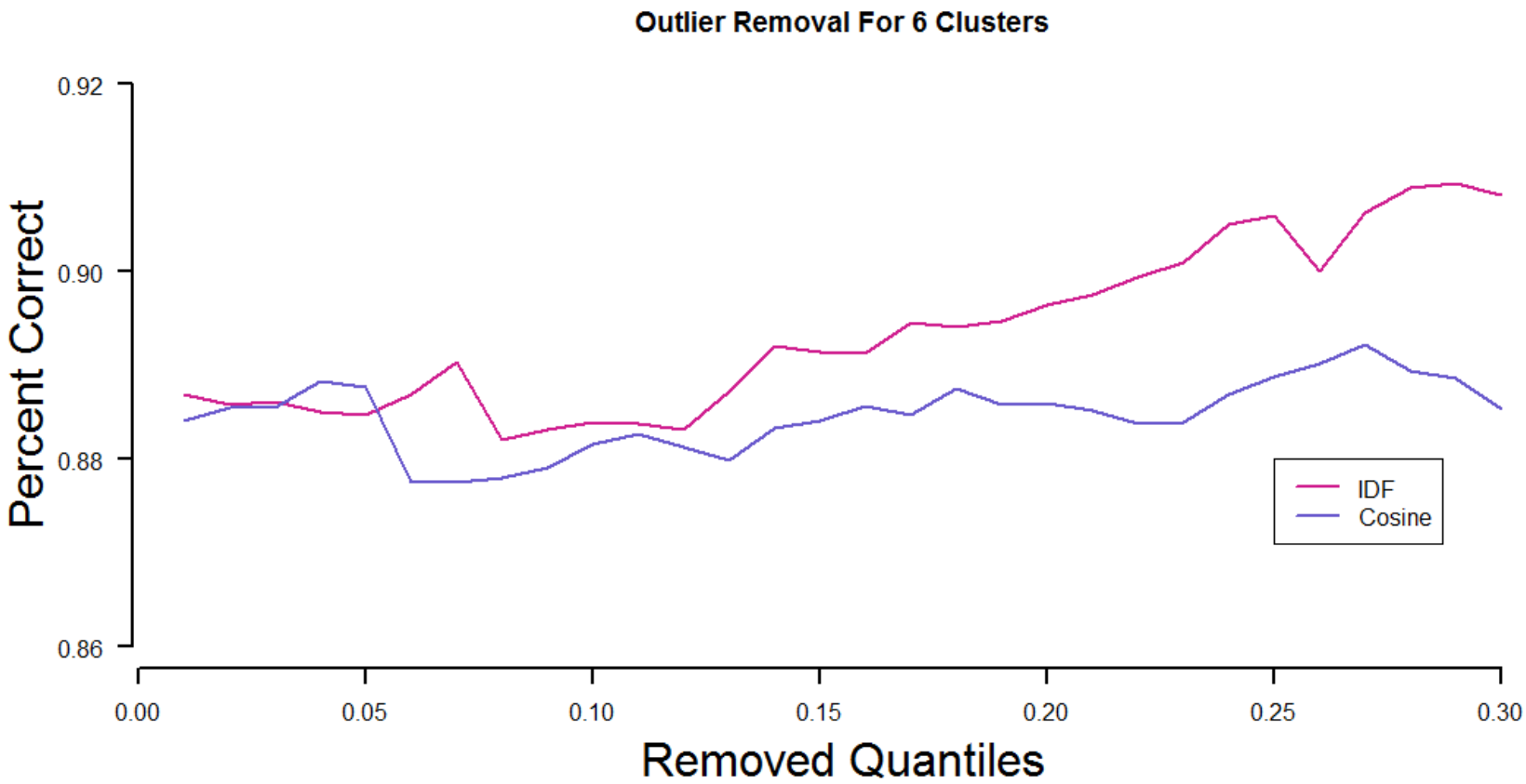


- Row sums of the cosine similarity matrix count as a measure of connectivity to other documents

Outlier Removal

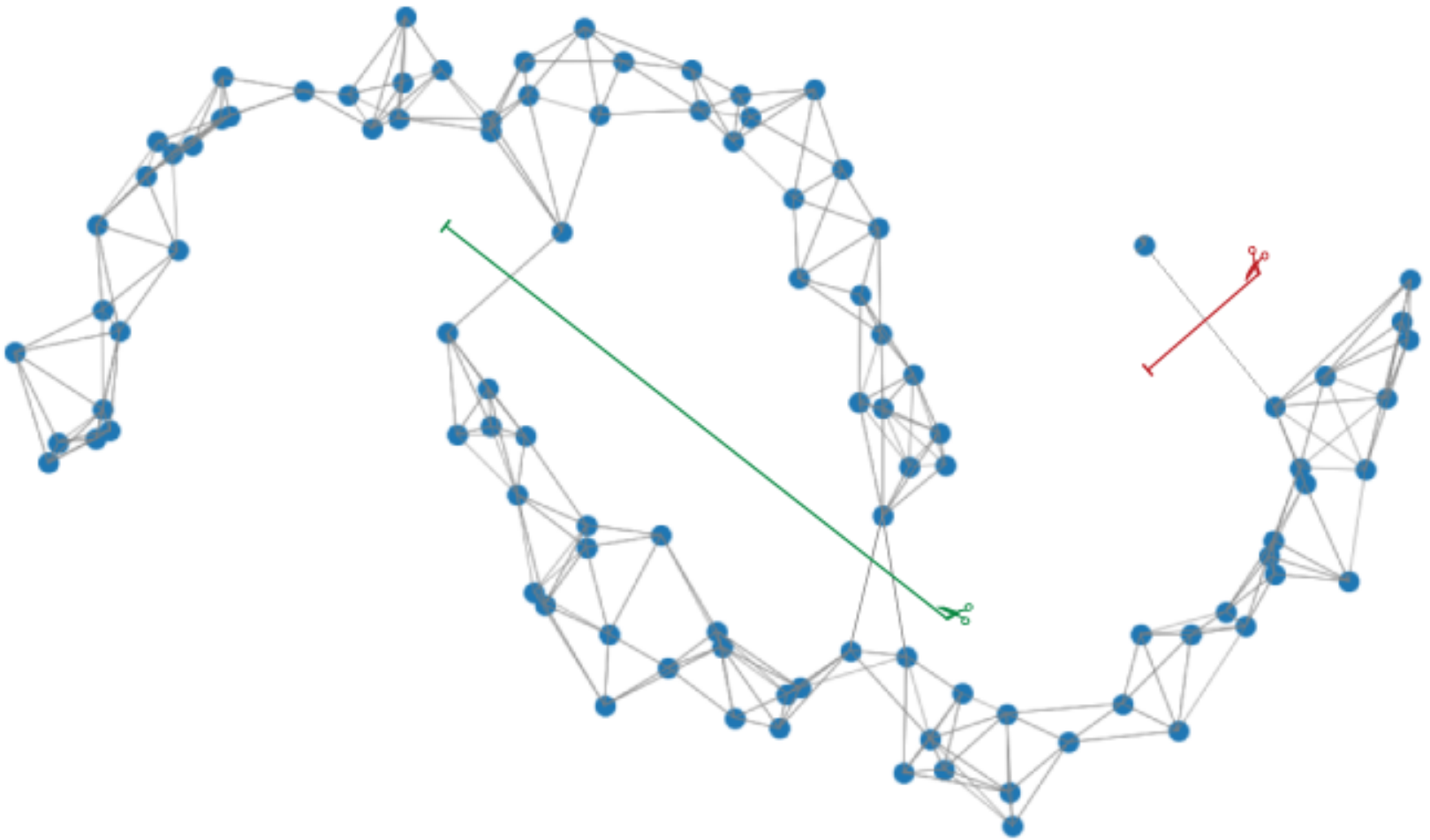


Outlier Removal



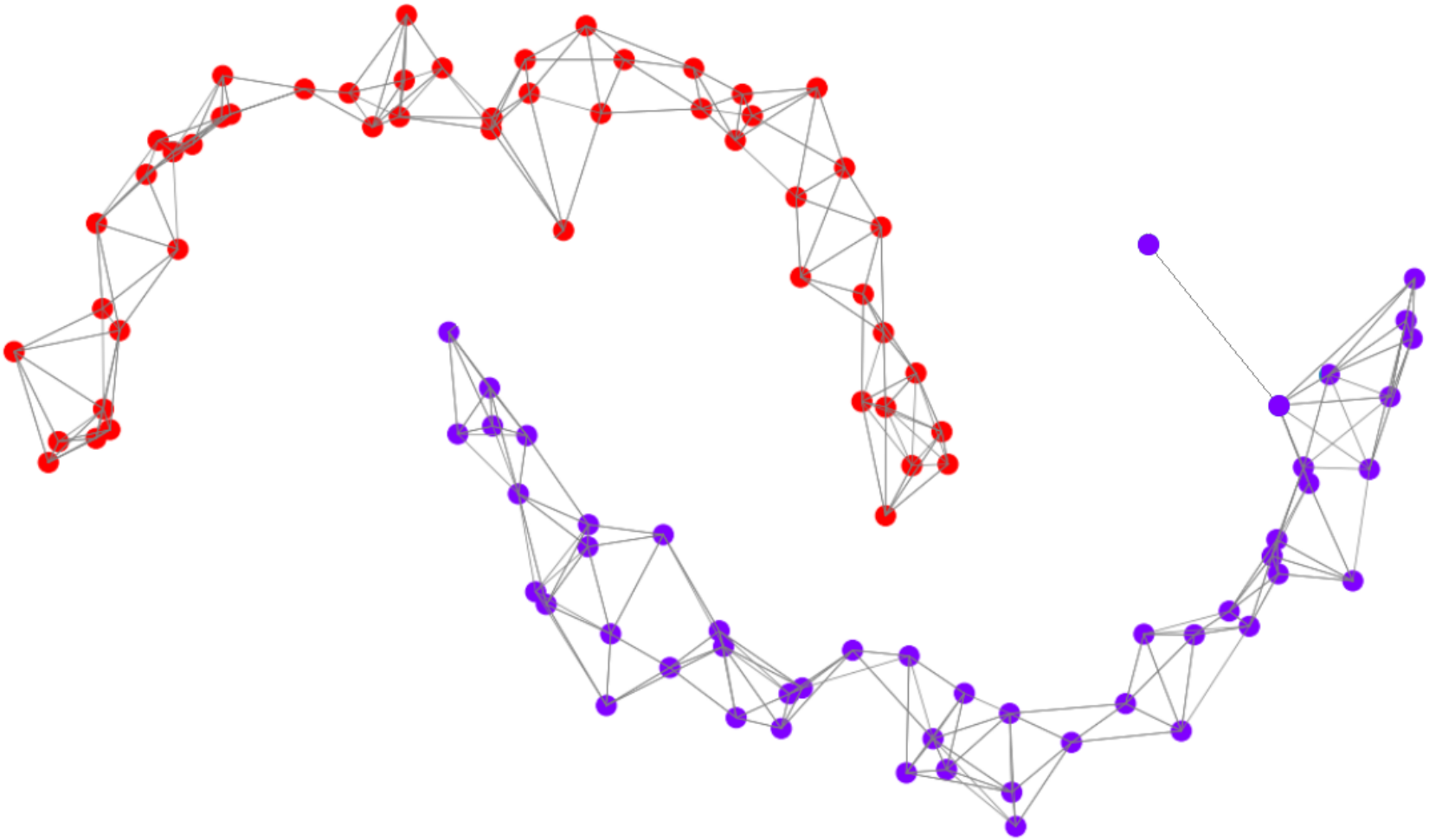
Spectral Clustering - Andrew

Spectral Clustering- Normalized Cut



$= \min Cut(A, B)$ $= \min \frac{Cut(A,B)}{Vol(A)} + \frac{Cut(A,B)}{Vol(B)}$

An ideal way to remove edges to create two clusters



Let D be a diagonal matrix where $D_{ii} = \sum_{j=1}^{n} w_{ij}$

Also let $x_i = \begin{cases} 1, \text{ if the observation i is in cluster A} \\ -1, \text{ otherwise} \end{cases}$

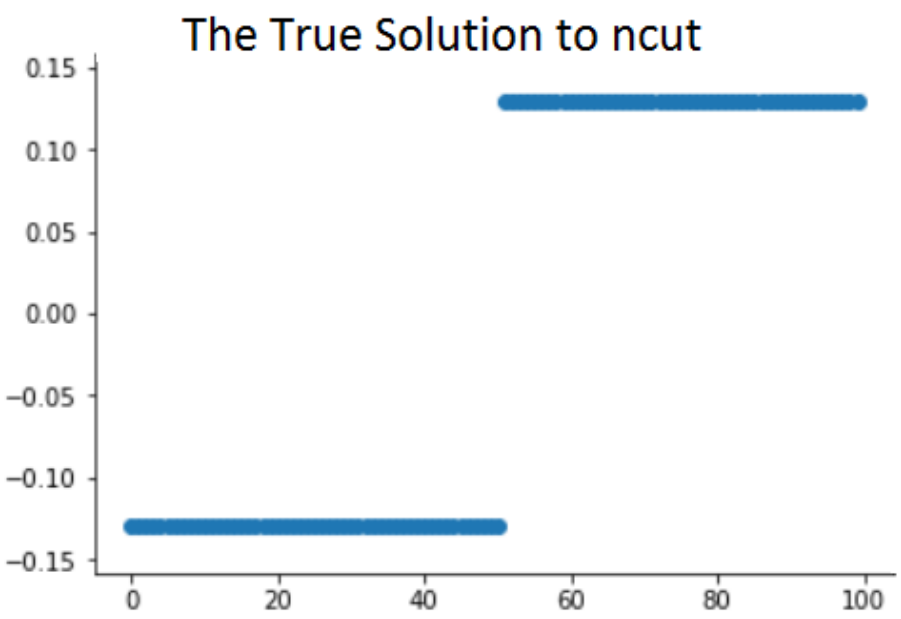
It can be shown that

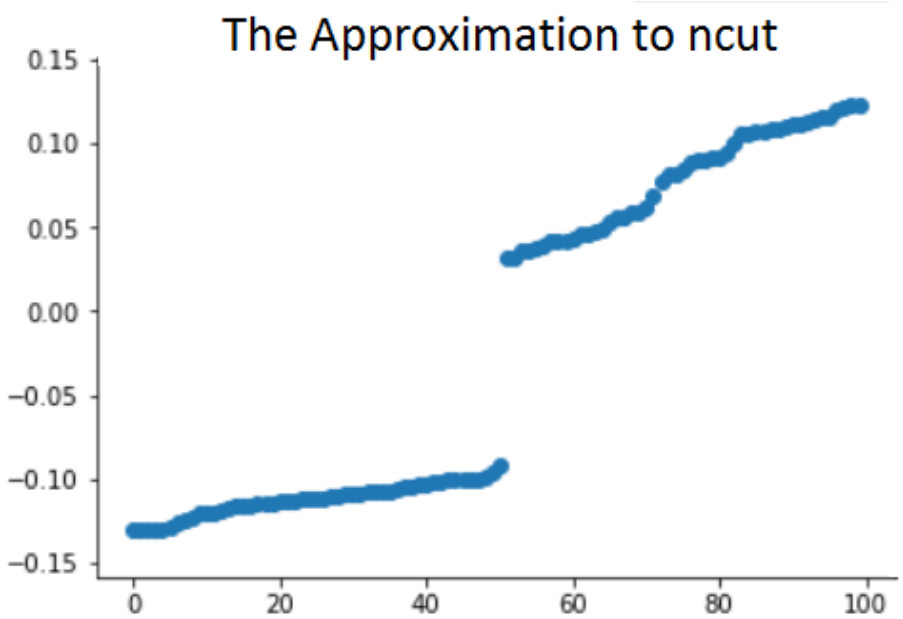
$$\min \left(\frac{Cut(A,B)}{Vol(A)} + \frac{Cut(A,B)}{Vol(B)} \right) = \min_{\mathbf{y}} \frac{\mathbf{y}^T \mathbf{L} \mathbf{y}}{\mathbf{y}^T \mathbf{D} \mathbf{y}}$$

Where $y = (1 + \mathbf{x}) - \frac{Vol(A)}{Vol(B)}(1 - \mathbf{x})$ and $\mathbf{L} = \mathbf{D} - \mathbf{W}$.

The solution to the normalized cut problem can be approximated by the sign of the second largest eigenvector of $\mathbf{D}^{-1}\mathbf{L}$

Spectral Clustering- Normalized Cut





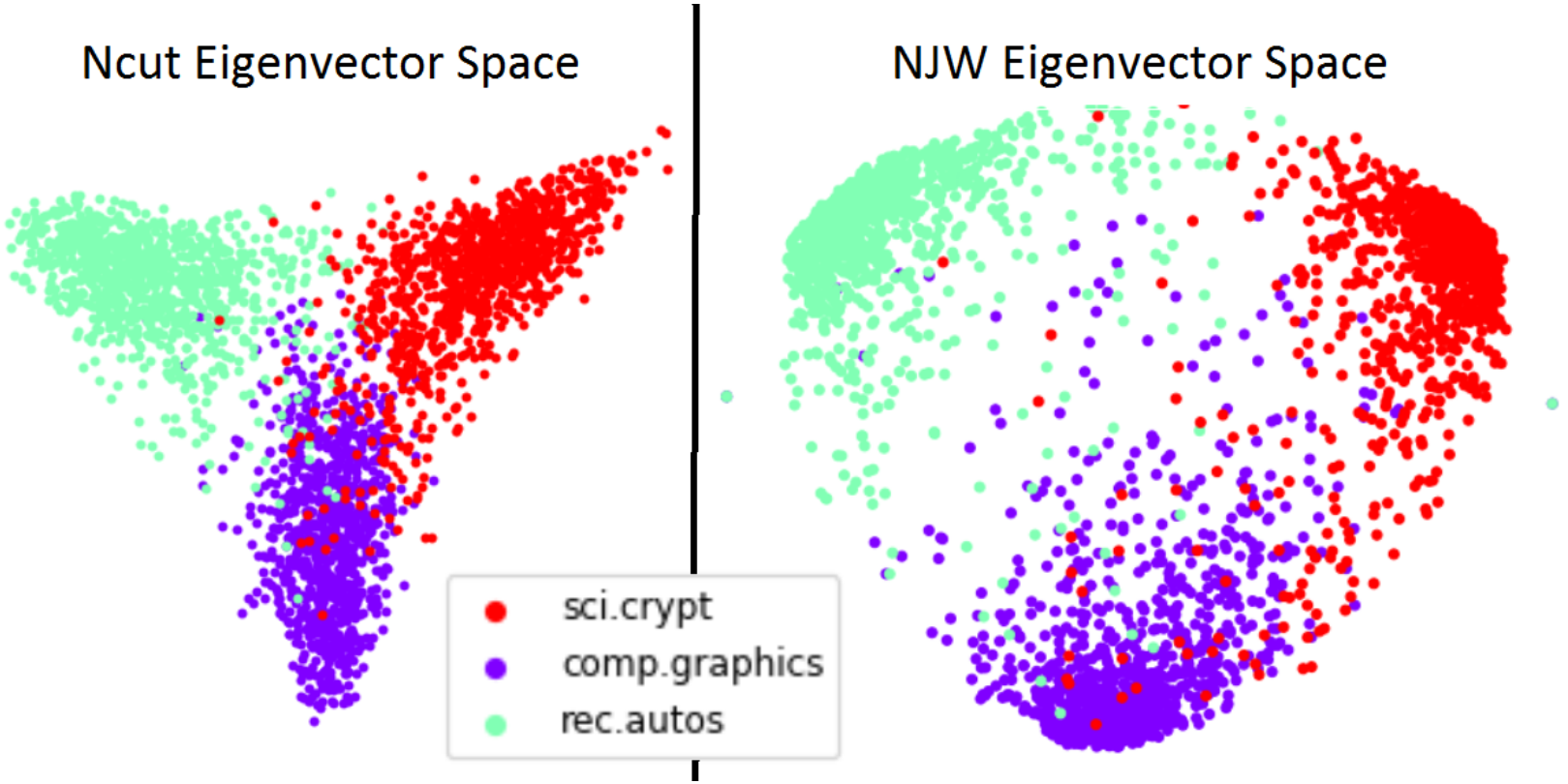
Algorithm (Ncut)

1. Construct similarity matrix $\mathbf{W}$
2. $\mathbf{L} = \mathbf{D} - \mathbf{W}$
3. Find the first k eigenvectors of $\mathbf{D}^{-1}\mathbf{L}$
4. Make a matrix $\mathbf{V}$ by stacking the 2nd to kth eigenvectors
5. Cluster using kmeans using $\mathbf{V}$ where each row represents a point

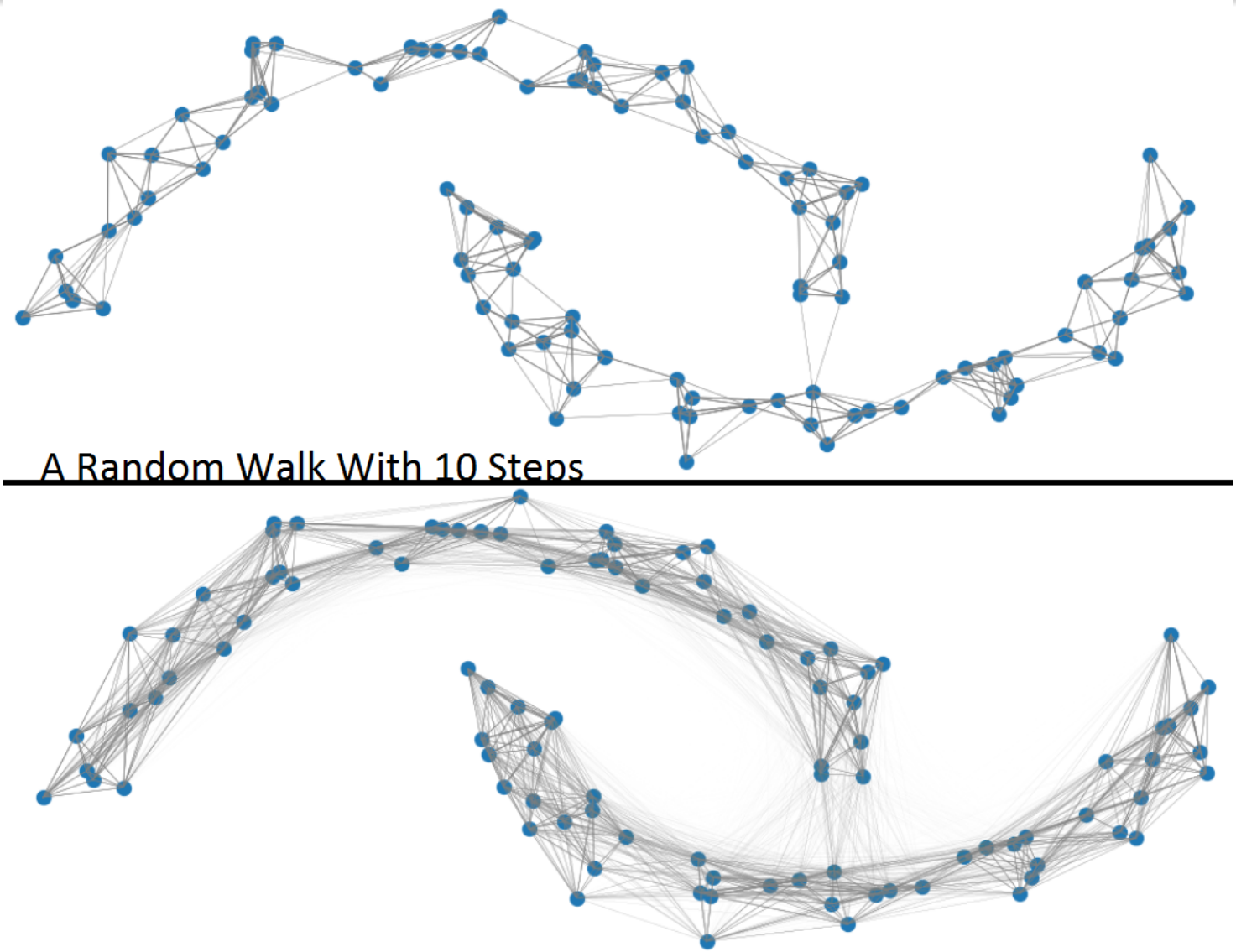
Algorithm (NJW)

1. Construct similarity matrix $\mathbf{W}$
2. $\mathbf{L} = \mathbf{D} - \mathbf{W}$
3. Find the first k eigenvectors of $\mathbf{D}^{-1/2}\mathbf{L}\mathbf{D}^{-1/2}$
4. Make a matrix $\mathbf{V}$ by stacking the first k eigenvectors
5. Normalize the rows of $\mathbf{V}$
6. Cluster using kmeans using $\mathbf{V}$ where each row represents a point

Spectral Clustering- Ng, Jordan, Weiss



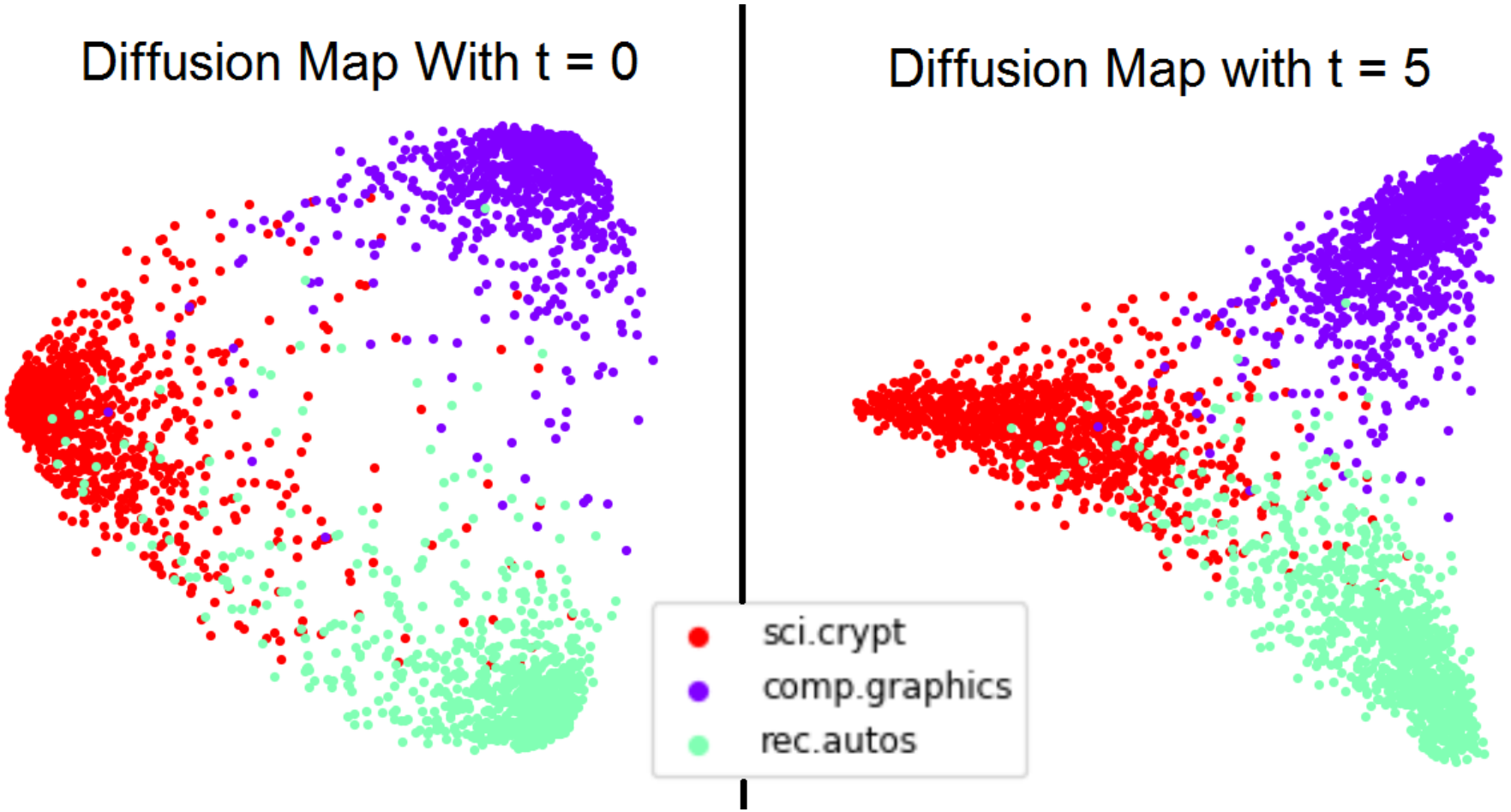
Spectral Clustering- Diffusion Map



Algorithm (Diffusion Map)

1. Construct similarity matrix $\mathbf{W}$
2. $\mathbf{L} = \mathbf{D} - \mathbf{W}$
3. Find the first k eigenvectors of $\mathbf{D}^{-1}\mathbf{L}$
4. Make a matrix $\mathbf{V}$ by stacking the 2nd to kth eigenvectors
5. Normalize the rows of $\mathbf{V}$
6. $V = (\lambda_1^t v_1, \lambda_2^t v_2, ..., \lambda_k^t v_k)$
7. Cluster using kmeans using $\mathbf{V}$ where each row represents a point

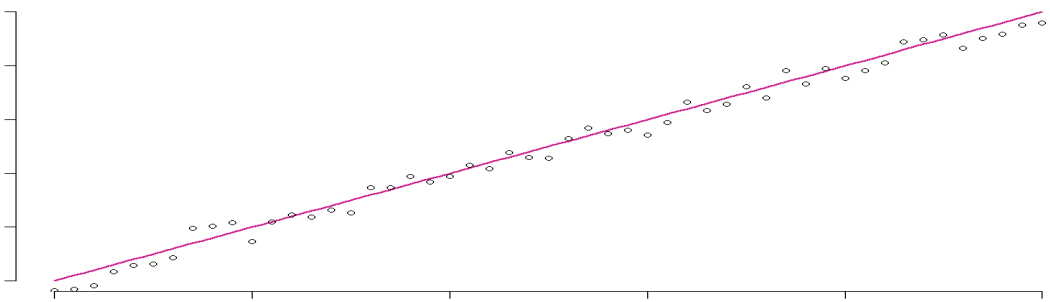
Spectral Clustering- Diffusion Map



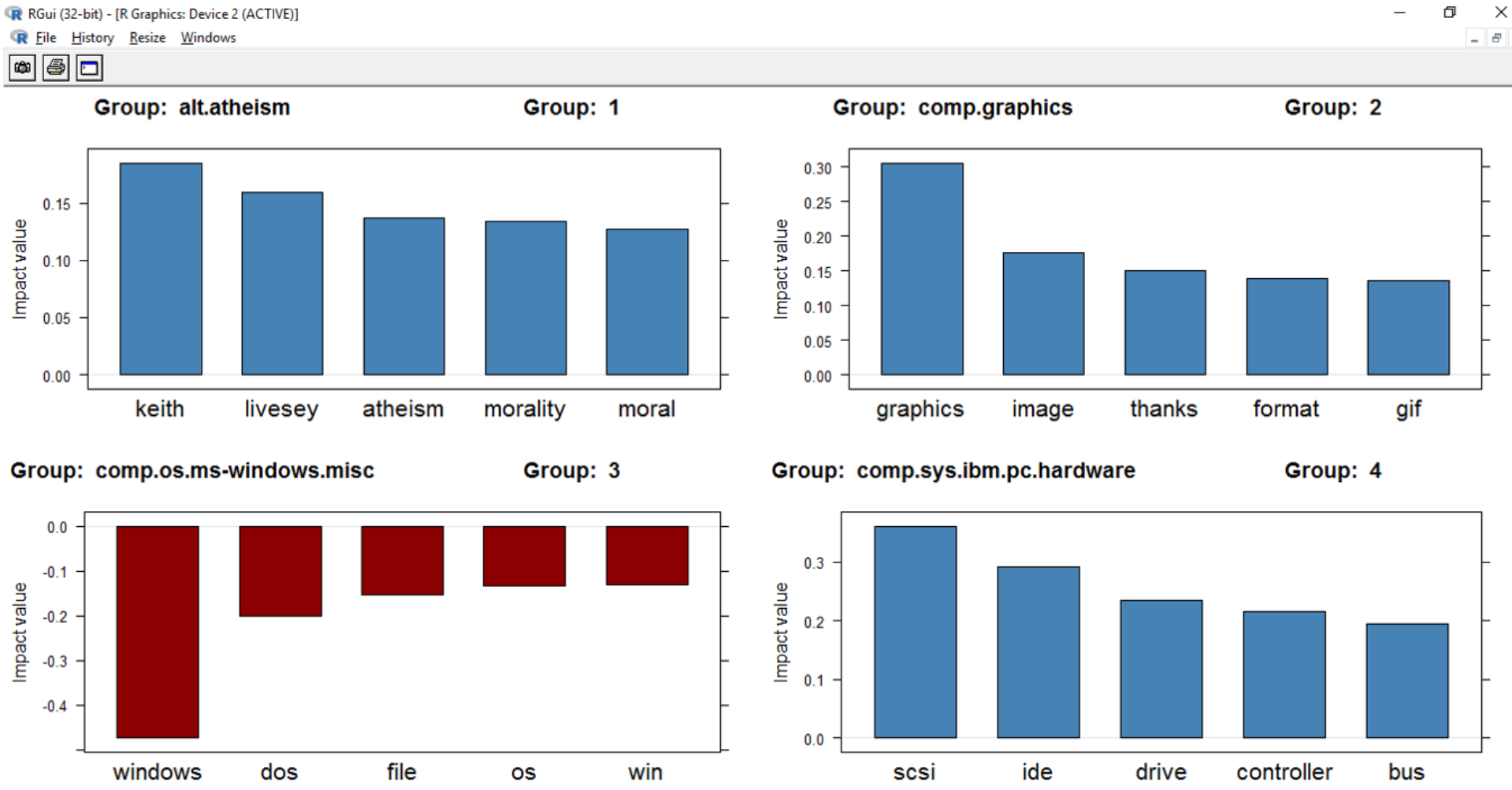
Clustering Insights - Nate

Insights

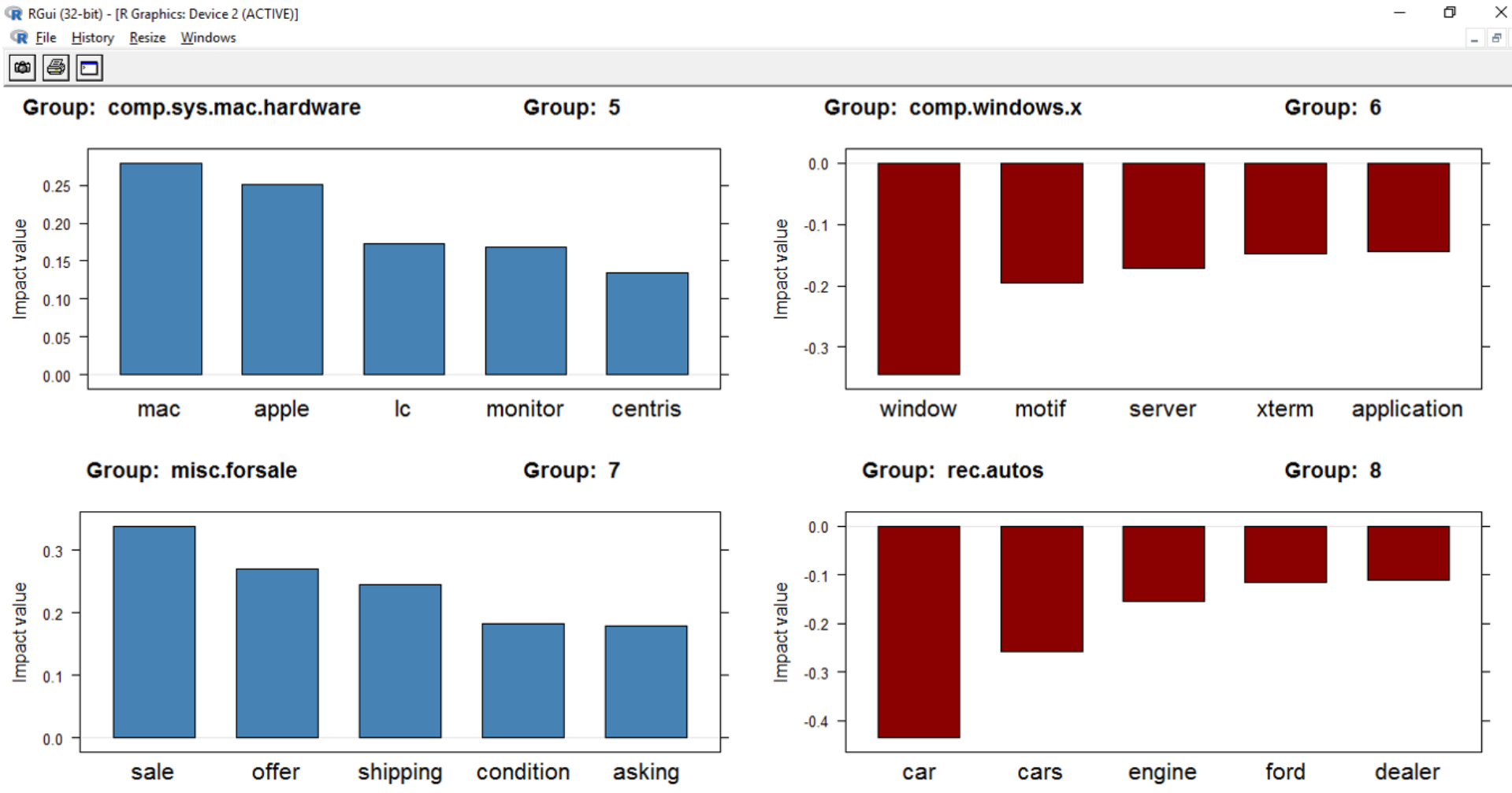
- We want to determine the columns that explain the principle "direction" of each cluster.
- This is done by finding the Principal Component vector after SVD



Cluster Insight



Cluster Insight



20 Newsgroup Clustering Results - Shiou-Shiou

Data and Tasks

comp.graphics
comp.os.ms-windows.misc
comp.sys.ibm.pc.hardware
comp.sys.mac.hardware
comp.windows.x

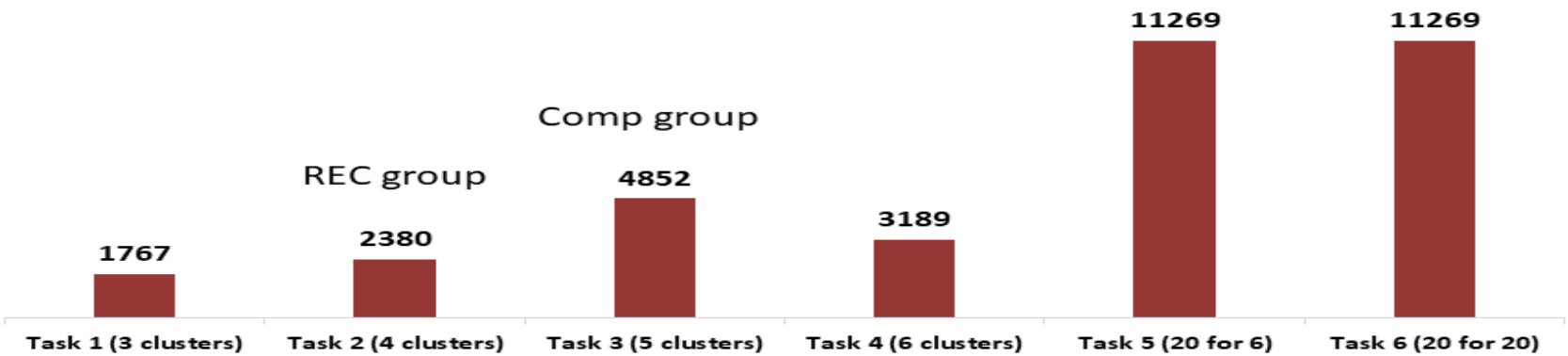
talk.**politics**.misc
talk.**politics**.guns
talk.**politics**.mideast

sci.crypt
sci.electronics
sci.med
sci.space

rec.autos
rec.motorcycles
rec.sport.baseball
rec.sport.hockey

talk.**religion**.misc
alt.**atheism**
soc.**religion**.christian

misc.forsale

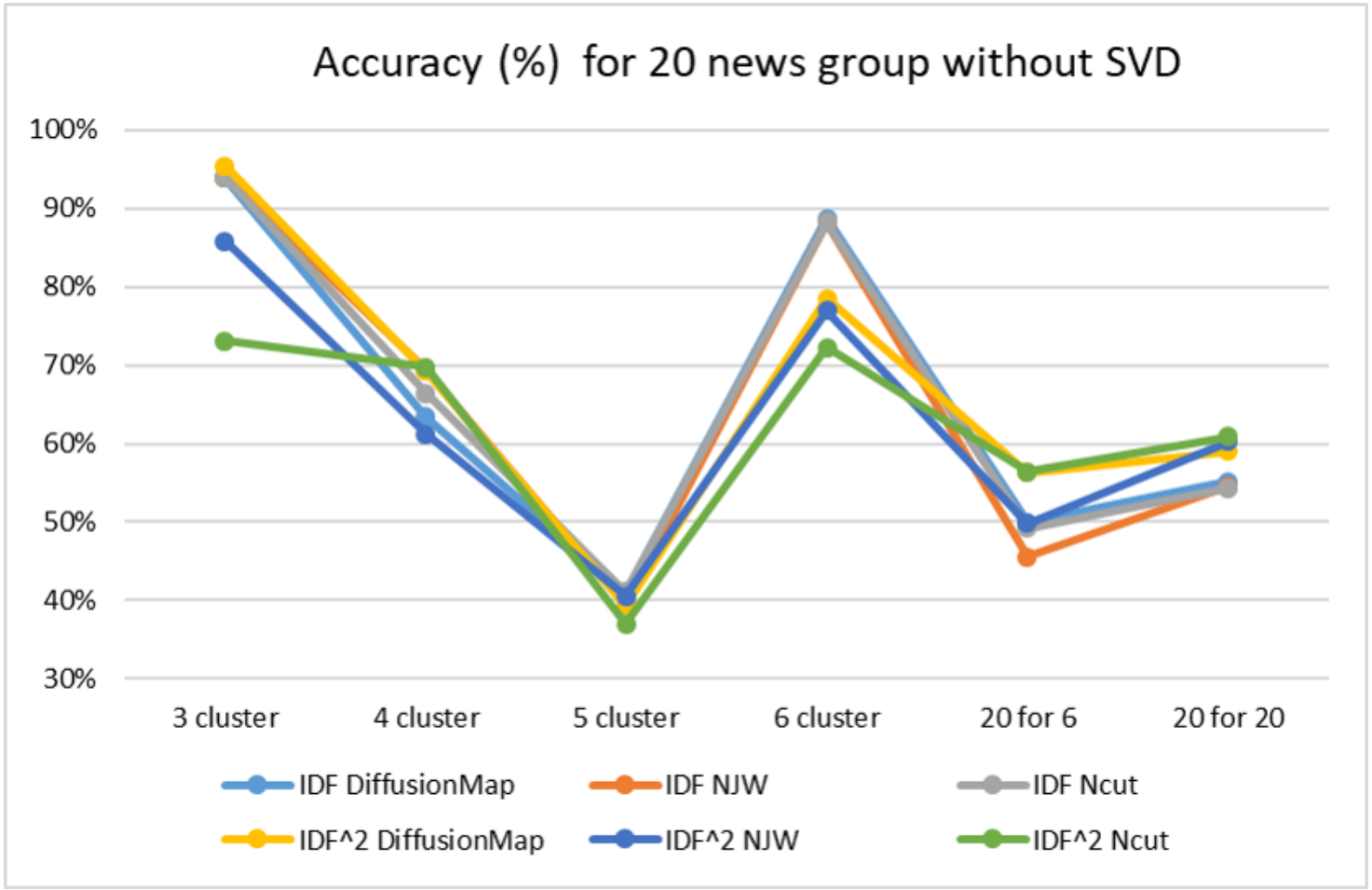


Measurements

- Accuracy: The percentage of data points that are truly in the same cluster are predicted to be in the same cluster.

- Adjusted Rand Index (ARI), F-measure,..

20 Newsgroup Clustering Results



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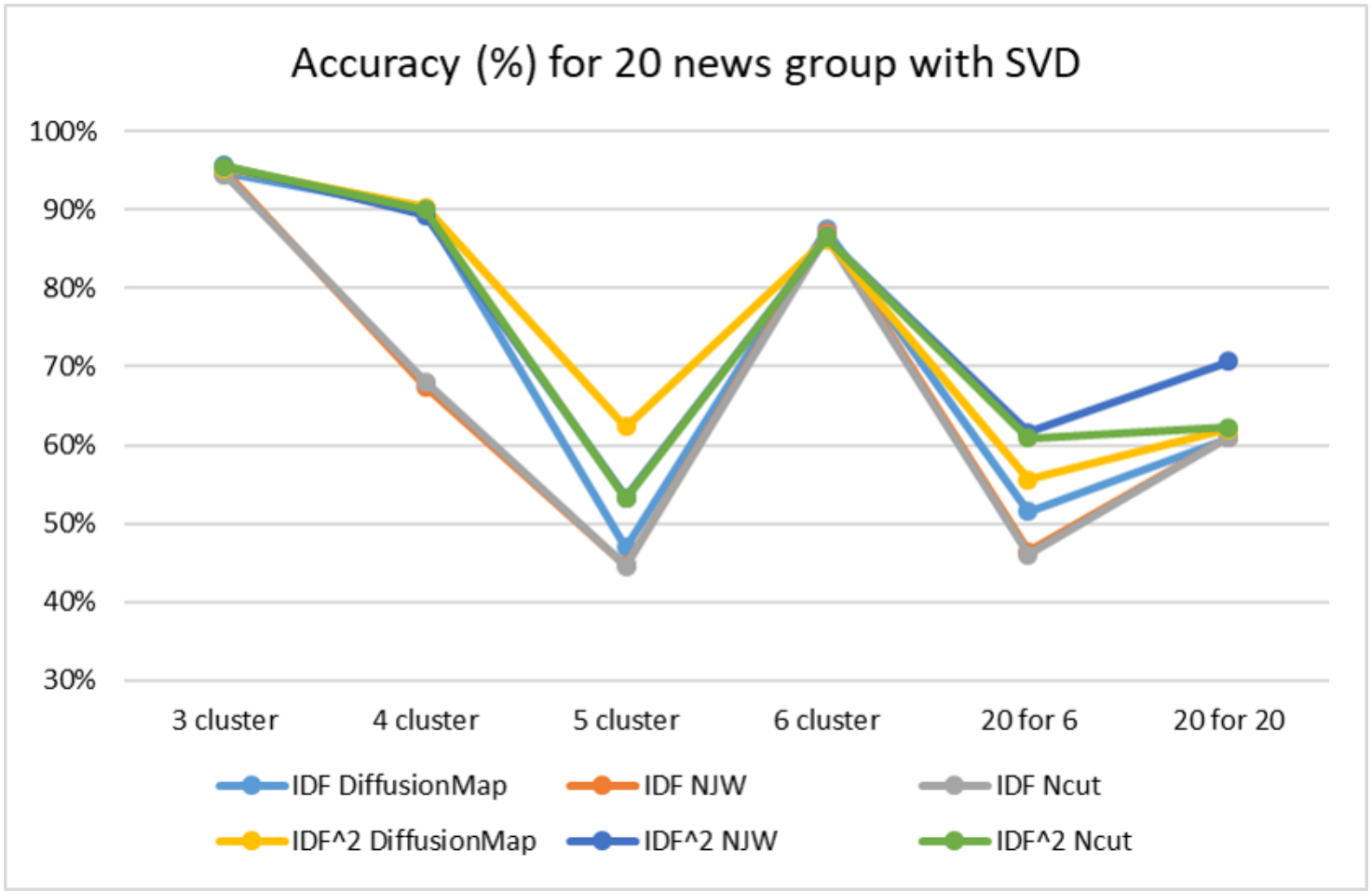
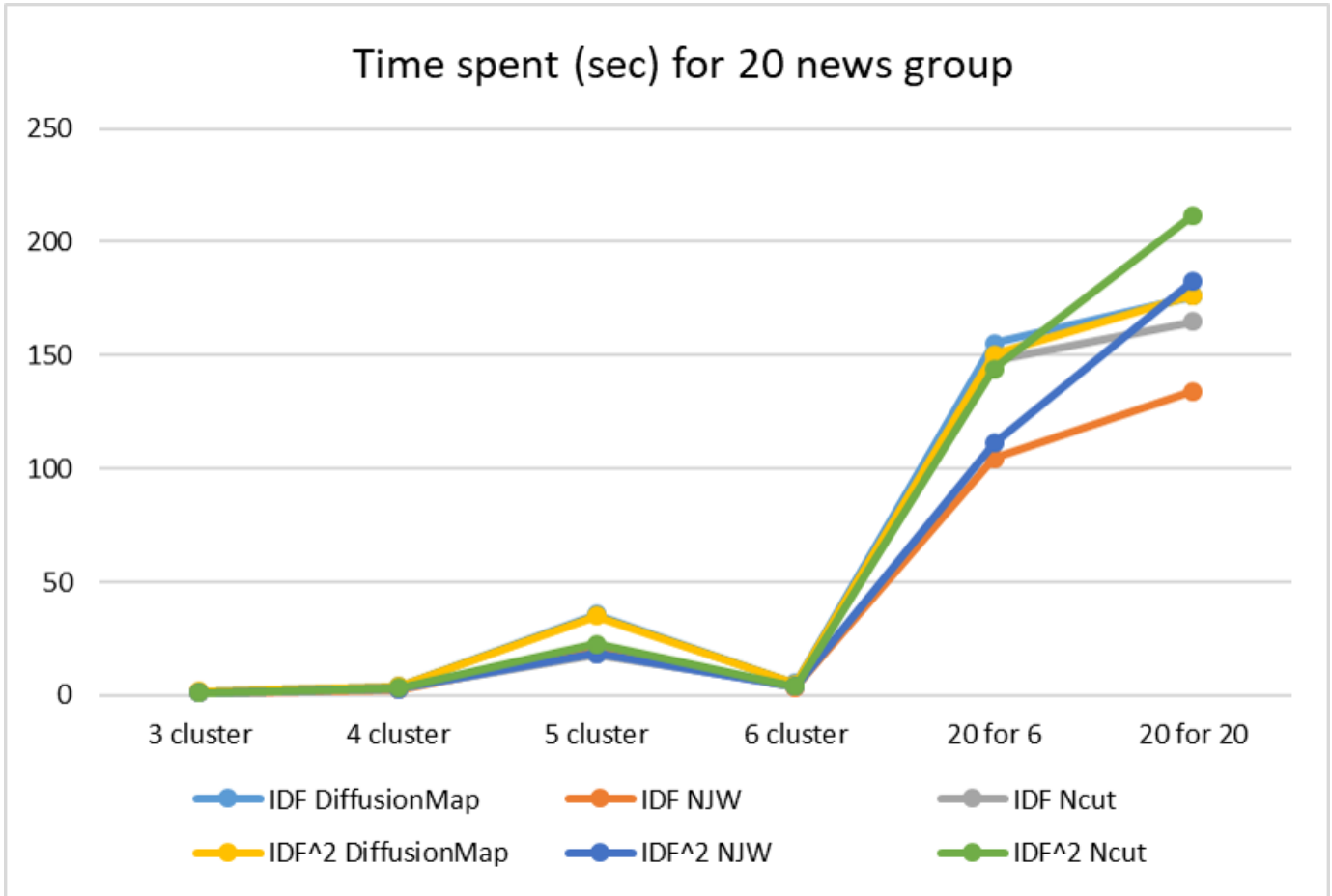


20 Newsgroup Clustering Results



Future Work - Joey

Efficiency Improvements: Landmark Centers

Sample Similarity Matrix

1	1	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	1	1	1

Randomly Selected "Centers"

1	1	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	1	1	1

Subset of Similarity Matrix

1	0	0
1	0	0
1	0	0
0	1	0
0	1	0
0	1	0
0	0	1
0	0	1
0	0	1

Efficiency Improvements: SVD Eigen-Estimation

- Normalized Similarity $= D^{-1/2} W D^{-1/2}$

$$= D^{-1/2}\Big(XX^{\mathsf{T}} - I_n\Big)D^{-1/2}$$

$$= \Big(D^{-1/2}X\Big)\Big(D^{-1/2}X\Big)^{\mathsf{T}} - D^{-1}$$

where $D = \text{RowSums}(W) = XX^{\mathsf{T}} \cdot \vec{1} = X\Big(X^{\mathsf{T}} \cdot \vec{1}\Big)$

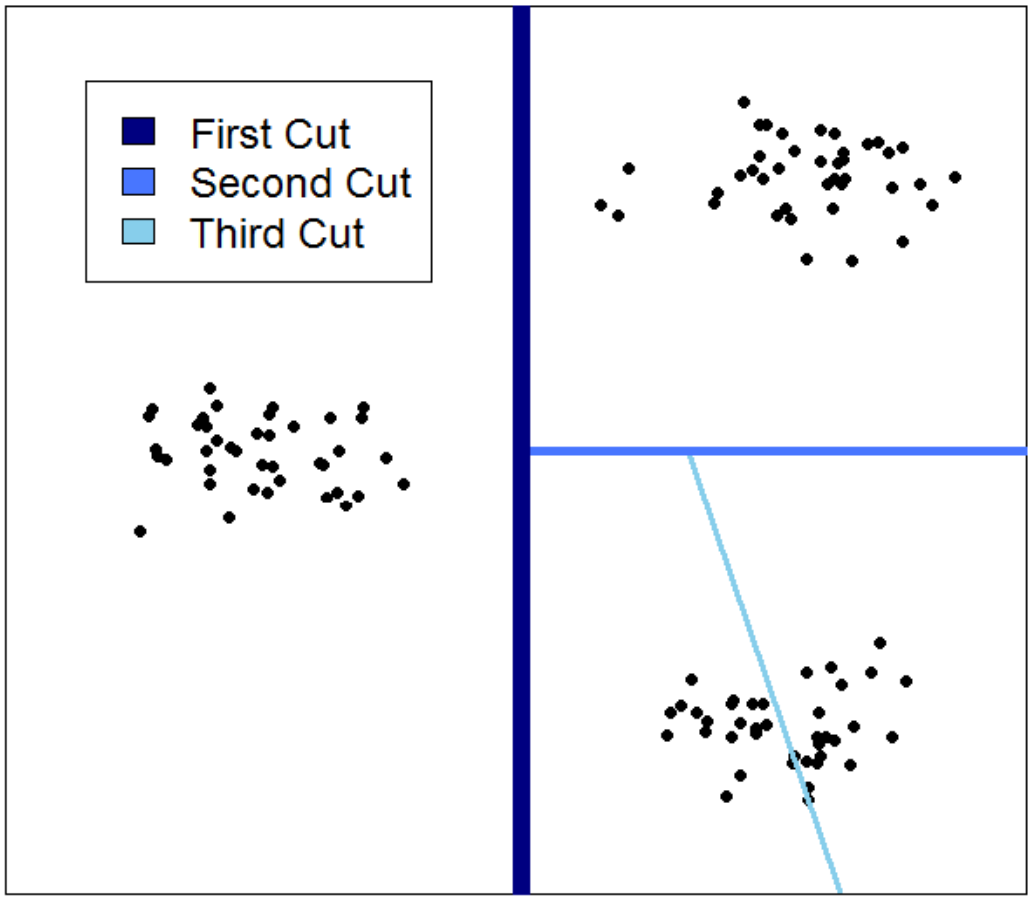
- $D \approx d \cdot I_n$:

$$\text{Eigenvectors}\left(D^{-1/2}XX^{\mathsf{T}}D^{-1/2} - D^{-1}\right) \Longleftrightarrow \text{S.V.D.}\Big(D^{-1/2}X\Big)$$

Adaptive Cluster Selection

- Divisive k-means

- Iterative Two-way Normalized Cuts



Feature Clustering

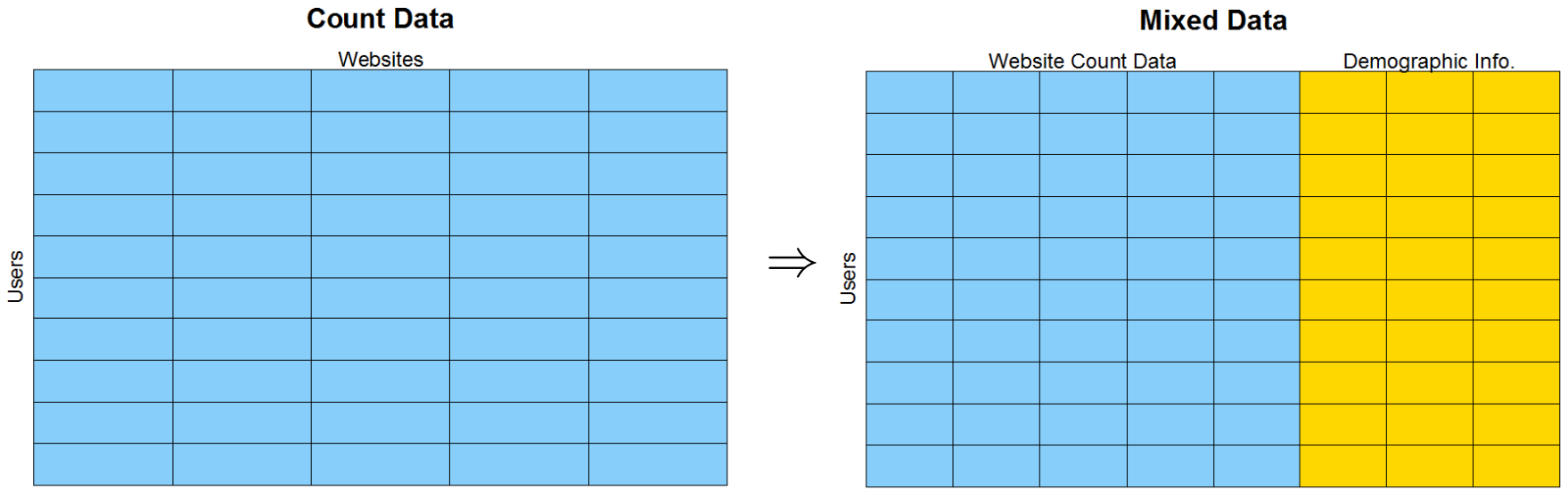
Plain Data Matrix

Documents \ Words									
	9	8	9	0	0	0	0	0	0
	8	5	8	0	0	0	0	0	0
	0	0	0	6	10	9	0	0	0
	0	0	0	7	8	6	0	0	0
	0	0	0	0	0	0	10	8	8
	0	0	0	0	0	0	7	6	10

Reduced Data Matrix

Documents	Word Group 1	Word Group 2	Word Group 3
	26	0	0
	21	0	0
	0	25	0
	0	21	0
	0	0	26
	0	0	23

Categorical Data



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Questions?