Hw0 (linear algebra review)

(1) Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. Answer the following questions (be sure to provide your reasoning):

(a) Is the above matrix $\mathbf{A}$ symmetric?
(b) Is the matrix $\mathbf{B} = \mathbf{A}^T\mathbf{A}$ symmetric?
(c) Is the matrix $\mathbf{C} = \frac{1}{2}(\mathbf{A} + \mathbf{A}^T)$ symmetric?
(d) What is $\mathbf{A}^2\,\mathbf{x}$, where $\mathbf{x} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$?

(2) Find the determinant of

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -4 & -3 \\ 2 & 3 & -7 & -6 \\ -1 & 0 & 6 & 4 \\ -4 & 9 & 9 & 8 \end{bmatrix}$$

Is the matrix invertible?

(3) Find a basis for each of the row and null spaces of

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 1 & 0 & 3 & 4 \\ 0 & 2 & -3 & 3 \end{bmatrix}$$

What is the rank of $\mathbf{A}$?

(4) For the matrix $\mathbf{A}$ specified in (3), find an orthonormal basis for its column space.

(5) Let

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Find its eigenvalues, corresponding algebraic and geometric multiplicities, and associated eigenspaces (by specifying a basis for each of them).

(6) For the matrix $\mathbf{A}$ defined in (5), is it diagonalizable? Orthogonally diagonalizable?

Depending on what your answer is, do one of the following accordingly:

- $\mathbf{A}$ is not diagonalizable: explain your reasoning why;
- $\mathbf{A}$ is only diagonalizable (and not orthogonally diagonalizable): explain why and find the diagonalization of $\mathbf{A}$;
- $\mathbf{A}$ is orthogonally diagonalizable: explain why and find the orthogonal diagonalization of $\mathbf{A}$.