# Worksheet 11: Eigenvalues and eigenvectors

**Example 0.74.** Let

$$\mathbf{A} = \begin{bmatrix} \frac{5}{2} & -1 \\ 1 & 0 \end{bmatrix}, \ \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Compute $\mathbf{A}\mathbf{v}_i$ for $i = 1, 2, 3$. Are they multiples of $\mathbf{v}_i$?

**Example 0.75.** Let

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Determine if $\mathbf{v}$ is an eigenvector of $\mathbf{A}$. If yes, find the corresponding eigenvalue.

**Example 0.76.** Determine if $-4$ is an eigenvalue of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$. If yes, find all eigenvectors associated to it.

**Example 0.77.** It is known that the following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

has two eigenvalues, 1 and 3. Find a basis for each of the two eigenspaces corresponding to them. What are the geometric multiplicities of the eigenvalues?

**Example 0.78.** The matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

is not invertible because 0 is an eigenvalue

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

**Example 0.79.** For each of the following matrices $\mathbf{A}$, first find an expression in $\lambda$ for $\det(\mathbf{A} - \lambda\mathbf{I})$:

(a) $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

and then use it to find the eigenvalues of $\mathbf{A}$ and corresponding algebraic multiplicities.

**Example 0.80.** Find the eigenvalues of

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

This is an example of a matrix that has complex eigenvalues.

**Example 0.81.** Determine the eigenvalues of the following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & & \\ & 2 & \\ & & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & & \\ 4 & 2 & \\ 5 & 6 & 3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 4 & 5 \\ & 2 & 6 \\ & & 3 \end{bmatrix}.$$