# Worksheet 8: Bases

**Example 0.54.** Show that the columns of $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ form a basis for $\mathbb{R}^2$.

**Example 0.55.** Determine if the columns of the matrix

$$\mathbf{A} = \begin{bmatrix} 3 & -4 & -2 \\ 0 & 1 & 1 \\ -6 & 7 & 5 \end{bmatrix}$$

form a basis for $\mathbb{R}^3$.

**Example 0.56.** Find a basis for the column space of

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**Example 0.57.** Find a basis for the column space of

$$\mathbf{B} = \begin{bmatrix} 1 & 4 & 0 & 2 & -1 \\ 3 & 12 & 1 & 5 & 5 \\ 2 & 8 & 1 & 3 & 2 \\ 5 & 20 & 2 & 8 & 8 \end{bmatrix}$$

**Example 0.58.** Find a basis for the null space of

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -2 & -5 & 7 & 5 \\ 3 & 7 & -8 & -5 \end{bmatrix}$$

**Example 0.59.** Consider the Euclidean space $\mathbb{R}^n$. Every vector $\mathbf{b} = (b_1, \ldots, b_n)^T$ in it has a unique representation under the standard basis:

$$\mathbf{b} = b_1\mathbf{e}_1 + \cdots + b_n\mathbf{e}_n$$

**Example 0.60.** We have previously showed that the columns of the matrix form a basis for $\mathbb{R}^3$:

$$\mathbf{A} = \begin{bmatrix} 3 & -4 & -2 \\ 0 & 1 & 1 \\ -6 & 7 & 5 \end{bmatrix}$$

Let $\mathbf{b} = [1\ 0\ 2]^T$. Find the unique set of scalars $c_1, c_2, c_3$ such that

$$\mathbf{b} = c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + c_3\mathbf{a}_3.$$