

Worksheet 1a: system of linear equations

Example 0.1. Solve the following linear system

$$\begin{cases} 2x_1 - x_2 + 5x_3 = 1 \\ x_1 - 2x_2 + 4x_3 = -1 \\ 3x_1 + x_2 + 6x_3 = 1 \end{cases}$$

Example 0.2. Solve the following modified linear system:

$$\begin{cases} 2x_1 - 4x_2 + 5x_3 = 1 \\ x_1 - 2x_2 + 4x_3 = -1 \\ 3x_1 - 6x_2 + 6x_3 = 1 \end{cases}$$

Example 0.3. Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$. Find $\mathbf{A}\mathbf{1}$ using both rowwise and columnwise methods. What does the product vector represent?

Example 0.4. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$$

Determine if $\mathbf{b} \in \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$, i.e., if \mathbf{b} is a linear combination of $\mathbf{v}_1, \mathbf{v}_2$.