The Influence of Atmospheric Stability on Potential Evaporation

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ABSTRACT

The Penman relationship for potential evaporation is modified to simply include the influence of atmospheric stability on turbulent transport of water vapor. Explicit expressions for the stability-dependent, surface exchange coefficient developed by Louis are used. The diurnal variation of potential evaporation is computed for the stability-dependent and original Penman relationships using Wangara data.

The influence of afternoon instability increases the aerodynamic term of the modified Penman relationship by 50% or more on days with moderate instability. However, the unmodified Penman relationship predicts values of daily potential evaporation close to that of the stability-dependent relationship. This agreement is partly due to compensating overestimation during nighttime hours. Errors due to use of daily-averaged variables are examined in detail by evaluating the nonlinear interactions between the diurnal variation of the variables in the Penman relationship.

A simpler method for estimating the exchange coefficient is constructed from an empirical relationship between the radiation Richardson number and the Obukhov length. This method is less accurate, but it allows estimation of the stability-dependent exchange coefficient using only parameters already required for evaluation of the Penman relationship. Finally, the diurnal variation of the atmospheric resistance coefficient appearing in the Penman–Monteith relationship is presented.

1. Introduction

The surface moisture flux is often parameterized in terms of potential evaporation and associated coefficient representing soil moisture deficit, resistance of the vegetation and radiative properties of the surface. The potential evaporation is defined as that evaporation occurring over a free water surface. In theory, the potential evaporation is independent of the state of the water surface and depends only on atmospheric conditions. In practice, the value of the potential evaporation depends on the methodology of measurement.

In most atmospheric models, the potential evaporation is parameterized in terms of a bulk aerodynamic relationship while in other disciplines the potential evaporation is more often equated to a Penman (1948) or combination formulation. The Penman formulation can be derived by combining the aerodynamic relationship with the surface energy balance. The Penman approach has the following advantages:

1) Surface temperature or surface saturation vapor pressure is eliminated. In practice, surface temperature is difficult to define over land where the difference between vegetation and soil temperatures can exceed 5°C. Associated errors in the bulk aerodynamic relationship have been shown to be large (Yu, 1977).

2) The Penman relationship includes an explicit dependence on net radiation which, when calibrated to actual evapotranspiration, may indirectly include biological dependencies on solar radiation such as photosynthesis. In fact, the frequently used Priestley–Taylor model (1972) expresses evapotranspiration exclusively in terms of net radiation.

3) The Penman relationship has been compared to actual evaporation or evapotranspiration in a large number of studies, although many studies are hard to compare due to use of different versions of the Penman equation, use of different observational levels, and influences of horizontal inhomogeneity.

In comparison with more empirical approaches, the Penman relationship is usually found to perform as well or better (e.g., Seguin, 1975). The Penman relationship has not been tested explicitly for downward moisture flux associated with negative net radiation and dew formation.

Evaluation of any model of potential evaporation from atmospheric variables, which are in equilibrium with a surface evaporating at less than potential, can be considered to be inconsistent (Bouchet, 1963; Morton, 1975; and others). Here we require the potential evaporation to be a function of only atmospheric variables and independent of reduction of actual moisture flux due to soil moisture deficit and plant resistance. However, the results of the present development can be transformed to modified expressions of potential evaporation.

The most serious disadvantage of the usual Penman relationship, and many other models of potential evaporation, is failure to include explicitly the influence of atmospheric stability on atmospheric transport of water vapor. Such an influence can significantly contribute to diurnal variation of the potential evaporation. The influence of stability can be reduced by using atmospheric variables measured closer to the ground. However, observations close to or within the canopy are difficult to interpret and the usual similarity theory no longer applies. As a result, studies of turbulent fluxes over land almost always include the influence of atmospheric stability.

However, only a few of the many applications of the Penman or combination formulations have included the influence of atmospheric stability. Such formulations are usually based upon similarity modifications of the log-law and thus also include a dependence on surface roughness in contrast to the original Penman relationship. Businger (1956) modified the Penman relationship to include a stability correction which was expressed in monogram form. Fuchs *et al.* (1969) included the influence of stability in a version of the combination equation which did not eliminate surface temperature. Federer (1970) included a stability adjustment in the Penman relationship which required knowledge of the Obukhov length or an additional unspecified relationship between stability parameters.

The Penman–Monteith (Monteith, 1965) relationship has been modified to include certain aspects of the stability influence using Obukhov similarity theory (Stewart and Thom, 1973; Thom and Oliver, 1977; Verma *et al.*, 1976; DeHeer-Amissah *et al.*, 1981; Berkowicz and Prahm, 1982). Such inclusion of the stability influence generally requires iterative procedures. Stricker and Brutsaert (1978) apply an iterative technique to estimate the stability parameter and its influence on the actual surface moisture flux as computed from the surface energy budget, while Brutsaert (1982) suggests an iterative procedure based on the Penman relationship. They conclude that the influence of atmospheric stability cannot be neglected, although they found little difference between the various stability formulations examined.

The first goal of this study is to present a method for estimating potential evaporation from the Penman relationship which includes the influence of atmospheric stability yet is simpler than the foregoing procedures. In particular, we wish to avoid the need for iteration in order to construct a method suitable for use in those atmospheric models or routine applications where computer time is restricted. This will be done by using the dependence of surface exchange coefficients on the bulk Richardson number presented in Louis (1979) and Louis *et al.* (1982) for both the stable and unstable cases (Section 2).

A second goal of this study is to estimate the importance of the influence of atmospheric stability since most applications of the Penman relationship still neglect such an influence. Toward this goal, we will systematically evaluate the Penman relationship with and without the stability influence using data from the Wangara experiment (Clarke *et al.*, 1971). The inclusion of the stability influence in this study is expected to improve substantially the Penman relationships since the Louis (1979) formulation approximates similarity theory which has been calibrated and tested against classical data sets. However, the modeled stability influence will incur some error so that the evaluation of the original Penman relationship will be only approximate.

Also of interest is the influence of the diurnal variation of stability on the 24 h evaporation since evaporation values are often reported for 24 hour periods. While afternoon instability can significantly enhance the surface moisture flux, nocturnal stability can significantly reduce such fluxes leading to some cancellation between stability influences. The third goal of this study is to assess the magnitude of errors resulting from use of 24 h averaged variables since the Penman relationship is frequently evaluated with such averaged data.

Note that this study is concerned only with potential evaporation dictated by atmospheric variables; no attempts are made to estimate the actual evaporation or relate it to the potential evaporation. The daily potential evaporation during the Wangara observational period averaged a little more than 2 mm day^{-1} reaching near 4 mm on some days. These modest values would be more than enough to evaporate the $2\frac{1}{2}$ cm of precipitation which fell during the 43 day period. Therefore, the actual evaporation rate was probably well below the potential rate for much of the observational period.

The study of the behavior of potential evaporation under conditions where the actual evaporation is less than potential, is of considerable interest. Plant–soil models, which are forced by expressions for potential evaporation, are typically applied to situations of moisture stress.

It should be noted that models or expressions which relate actual evaporation to potential evaporation depend largely on observational calibration. Consequently, improved physical basis sought in this study will not necessarily lead to improved prediction of evapotranspiration in field situations. The relationship between actual and potential evaporation is outside the scope of this development. However, the procedures presented here will allow future construction of simple, physically more consistent, models of interaction between evaporation and the atmosphere.

2. Basic development

We begin with the bulk aerodynamic relationships for surface moisture and temperature flux:

$$(\overline{w'q'})_{\text{sfc}} = C_q u(q_{\text{sfc}} - q), \tag{1}$$

$$(\overline{w'T'})_{\text{sfc}} = C_h u(T_{\text{sfc}} - T), \tag{2}$$

where u, q, and T are, respectively, the atmospheric wind speed, specific humidity and temperature measured at a standard level such as 2 m, C_q and C_h are nondimensional exchange coefficients, the subscript "sfc" refers to surface values, w is the vertical motion and primes indicate turbulent fluctuations. Relationships (1) and (2), in principle, assume that the mean flow is sufficiently horizontally homogeneous so that turbulence is uniquely in equilibrium with the mean flow.

The exchange coefficient appearing in the bulk aerodynamic relationship can vary by several factors, with only modest diurnal variations of atmospheric stability. The same can be said of the coefficient appearing in Dalton's law used to derive the original Penman formula.

The potential evaporation can be defined by replacing q_{sfc} in (1) with the saturation surface value q^*_{sfc} corresponding to the temperature of the surface, in which case

$$\overline{(w'q')}_{sfc} = C_q u(q^*_{sfc} - q). \quad (3)$$

In analogy with the usual Penman derivation, we expand (3) so that

$$\overline{(w'q')}_{sfc} = C_q u[(q^*_{sfc} - q^*) + (q^* - q)], \quad (4)$$

where q^* is the saturation value of the atmospheric specific humidity at the standard observational level.

To continue the analogy, we express the saturation specific humidity as a function of temperature. Using the relationship

$$q^* \approx 0.622 \frac{e^*}{p},$$

we obtain the approximation

$$q^*_{sfc} - q^* = \frac{0.622}{p} \frac{de^*(T)}{dT} (T_{sfc} - T), \quad (5)$$

where e^* is the saturation vapor pressure and p is atmospheric pressure. Substituting (5) into (4), we obtain

$$\overline{(w'q')}_{sfc} = C_q u\left[\frac{0.622}{p} \frac{de^*(T)}{dT} (T_{sfc} - T) + q^* - q\right]. \quad (6)$$

Using (2), we can write the surface energy balance as

$$-\rho L_v \overline{(w'q')}_{sfc} + R_n - S - \rho c_p C_h u(T_{sfc} - T) = 0, \quad (7)$$

where ρ is the surface density, L_v the specific latent heat, c_p the specific heat capacity, R_n the net radiative energy gained by the surface and S the flux of heat to the soil or vegetation, R_n and S are expressed in W m^{-2}.

To eliminate surface temperature we combine (6) and (7) and obtain

$$E = \frac{\Delta(R_n - S)}{C_h/C_q + \Delta} + \frac{\rho L_v C_q u(q^* - q)}{(1 + (C_q/C_h)\Delta)}, \quad (8)$$

where

$$\left.\begin{aligned} E &\equiv \rho L_v \overline{(w'q')}_{sfc} \\ \Delta &\equiv \frac{0.622}{p} \frac{L_v}{c_p} \frac{de^*(T)}{dT} \end{aligned}\right\},$$

and E is the potential latent heat flux at the surface in W m^{-2}. Equivalently, the evaporation of water in mm day^{-1} is $3.46 \times 10^{-2}E$. The surface moisture flux $\overline{w'q'}$ in m s^{-1} g kg^{-1} is $10^3E/(\rho L_v) = 3.1 \times 10^{-4}E$.

With no other information, we assume that the turbulent exchange coefficients for heat and moisture are equal. Then (8) becomes

$$E = \frac{\Delta(R_n - S)}{1 + \Delta} + \frac{\rho L_v C_q u(q^* - q)}{1 + \Delta}. \quad (9)$$

Relationship (9) is similar to other Penman formulations except that the usual wind function $f(u)$ is replaced by $C_q u$.

The second term is normally referred to as the advection term, since with no mean wind speed and no evaporation the specific humidity may approach saturation. However, even in the theoretical limit of vanishing wind speed, turbulence generated by any surface heating will mix drier air downward, keeping air near the surface unsaturated. As in the original Penman relationship, the second term in (9) does not vanish with vanishing wind speed if the air is not saturated and if the dependence of C_q on stability is appropriately chosen. The same can be said of the second term when it is identified with the evaporation measured by a Piché atmometer (Brochet and Gerbier, 1972). Even as the wind speed vanishes, convectively driven turbulence can ventilate the atmometer, leading to evaporation. For lack of a better term, we will refer to the second term as the "aerodynamic" term although it must be remembered that both terms in (9) originate from the bulk aerodynamic relationship.

The original Penman relationship is derived in the same manner as (9) except Dalton's law is used as a starting point in place of (1). This relationship is typically expressed in the form (e.g., Brutsaert, 1982)

$$\left.\begin{aligned} E &= \frac{\tilde{\Delta}}{\tilde{\Delta} + \gamma} \frac{Q}{L_v} + \frac{\gamma}{\tilde{\Delta} + \gamma} f(u)(e^* - e) \\ f(u) &= 0.26(1 + 0.54u) \\ \tilde{\Delta} &\equiv \frac{de^*(T)}{dT} \\ \gamma &= \frac{C_p p}{0.622 L_v} \end{aligned}\right\}, \quad (10)$$

where E here is expressed in mm day^{-1}, Q is the net radiation flux density less soil heat flux, and e^* and e are, respectively, the saturation and actual values of vapor pressure at 2 m. The wind function $f(u)$ was determined from evaporation pan measurements (Penman, 1948). Many modifications of the Penman wind function have been suggested, although the original form still enjoys widespread usage. Comparison of (10) and (9) indicates that the wind function is proportional to the exchange coefficient

$$f(u) \propto C_q u,$$

where the coefficient of proportionality depends on the units employed in (1) and (10), (Brutsaert, 1982).

3. Dependence of exchange coefficient on stability

The dependence of the exchange coefficient C_q on atmospheric stability can be expressed in terms of a Richardson number of the form

$$\mathrm{Ri} = \frac{g}{\theta}\frac{(\theta - \theta_{\mathrm{sfc}})z}{u^2},$$

where g is the acceleration of gravity, z the height of the atmospheric observations, θ the atmospheric potential temperature and θ_{sfc} the potential temperature of the air at the lower reference level. The application of Monin–Obukhov similarity theory to the bulk aerodynamic relationship requires integration between two reference levels. The lower reference level is typically chosen to be the roughness height which simplifies the bulk aerodynamic relationship. The derivation of the Penman relationship demands integration from the surface where saturation is assumed in order that the vapor pressure can be determined from the temperature. The appropriate integration constant is then the roughness length for moisture (Brutsaert, 1982). Because of the approximate nature of this development and most applications to actual data, we will not distinguish between the roughness lengths for momentum and scalar quantities. For the present data analysis, the influence of water vapor on buoyancy is generally small and therefore also neglected.

Based on previous observations and certain asymptotic constraints, the development in Louis (1979), together with modifications in Louis *et al.* (1982), leads to the following dependence for the unstable case (Ri < 0)

$$C_q = \left[\frac{k}{\ln\left(\frac{z+z_0}{z_0}\right)}\right]^2\left(1 - \frac{15\,\mathrm{Ri}}{1 + C[-\mathrm{Ri}]^{1/2}}\right), \quad (11a)$$

where

$$C = \frac{75k^2\left(\frac{z+z_0}{z_0}\right)^{1/2}}{\left[\ln\left(\frac{z+z_0}{z_0}\right)\right]^2} \quad \text{for} \quad k = 0.4$$

and stable case (Ri > 0)

$$C_q = \left[\frac{k}{\ln\left(\frac{z+z_0}{z_0}\right)}\right]^2 [(1 + 15\,\mathrm{Ri})(1 + 5\,\mathrm{Ri})^{1/2}]^{-1}. \quad (11b)$$

Both formulations reduce to the usual log-law as $\mathrm{Ri} \rightarrow 0$. Note that the exchange coefficient increases with increasing instability (increasing negative Richardson number). In the limit of extreme instability ($\mathrm{Ri} \rightarrow -\infty$), after substituting for the Richardson number into (11a), the wind function becomes

$$C_q u \rightarrow \frac{2}{15}\left(\frac{z_0}{z+z_0}\right)^{1/2}\left[\frac{g}{\theta}(\theta_{\mathrm{sfc}} - \theta)z\right]^{1/2}. \quad (12)$$

Thus, the evaporation rate becomes independent of wind speed and depends on surface heating through a square root dependence on the surface temperature excess.

In the free convection limit, the roughness length becomes a somewhat arbitrary lower limit to the integration, which allows smooth matching to the usual Monin–Obukhov similarity theory. Ideally the expected rapid increase of $\theta_{\mathrm{sfc}} - \theta$ with increasing z/z_0 is such that the free convection limit (12) becomes independent of the roughness length. However, the actual sensitivity of (12) to the roughness length cannot be determined in practice since the air temperature becomes extremely inhomogeneous at the surface.

Other attempts to include the free convection limit involve additional criteria and separate formulation of the free convection case. These criteria could be added to the above development. However, the free convection limit is not usually of practical importance; e.g., in the Wangara experiment −Ri rarely exceeded one. As is evident from Fig. 1 of Louis (1979) for such stability limits, the modeled influence of roughness length on the stability correction to the exchange coefficient (11) is well-behaved in that it closely approximates the original fit to data by Businger *et al.* (1971). Consequently, for simplicity, we proceed with (11) without further modification.

The stability corrections in the foregoing exchange coefficient for heat and water vapor are different than that for momentum. However, the differences between the neutral values of the exchange coefficients for momentum and heat or moisture have been neglected in the formulation based on Louis *et al.* (1982). This contrasts to neutral limits in Louis (1979) and others where the exchange coefficient for heat was larger than momentum and also contrasts with Stewart and Thom (1973) where the relationship between the exchange coefficients was more complicated.

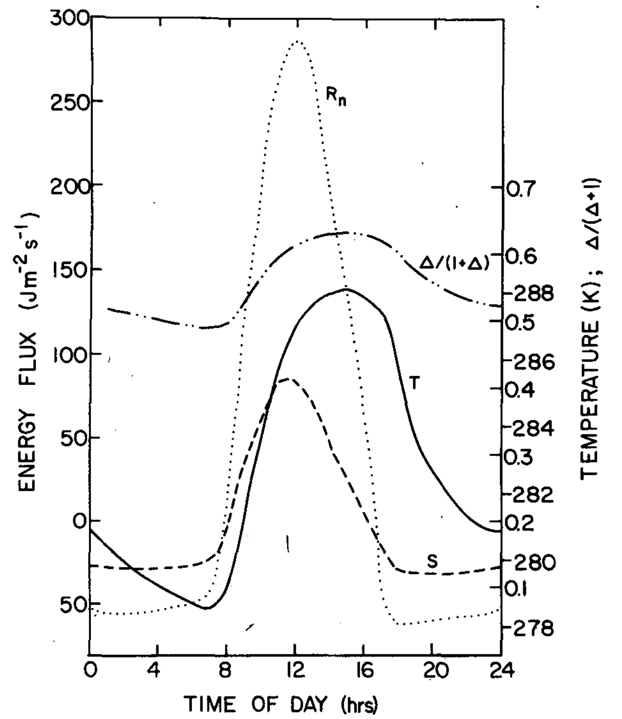

FIG. 1a. The mean diurnal variation of net radiation (dotted), heat flux to the soil (broken), temperature (solid) and coefficient of the radiation term $\Delta/(\Delta + 1)$ (dot-dash).

4. Asymptotic cases

We now identify various special or limiting cases where the wind speed, exchange coefficient, humidity deficit and/or net radiation less soil heat flux vanish (Table 1). Combinations which do not satisfy the surface energy balance have been eliminated. As before, molecular diffusion of vapor is not considered. The type of evaporation has been classified as free convection if the wind speed vanishes and the implied turbulence and vapor transport are driven only by buoyancy. Conversely, the evaporation is classified as mechanical if the sensible heat flux is zero or downward, in which case vapor is transported only by shear-generated turbulence.

Both the original and modified expressions agree on the existence of evaporation or dewfall for the various cases except for cases 7 and 8. Since the original Penman wind function contains no dependence on stability, its aerodynamic term incorrectly predicts free convection of water vapor away from the surface for vanishing wind speed under stable conditions (7a and 8a) when there should be no turbulence. The same problem would theoretically occur with vanishing wind speed and exactly neutral conditions. With the stability-dependent exchange coefficient, free convection of water vapor correctly occurs only for unstable conditions (upward heat flux).

5. Diurnal variations in Wangara

We now compute the diurnal variation of potential evaporation from micrometeorological data collected during the Wangara experiment near Hay, Australia, in the winter of 1967 (Clarke *et al.*, 1971). The diurnal variation of stability during the Wangara experiment was, on the average, less than most would expect. Except for day 33, the magnitude of the afternoon Obukhor length was generally near or greater than 10 m and on many afternoons greater than 100 m. This only modest instability is due to relatively low winter sun angles and generally significant airflow.

Potential evaporation is calculated from Wangara data using both the original Penman equation and the Penman equation modified to include the stability-dependent exchange coefficient. The 40 days of data provided by the Wangara program allow nearly 900 hourly calculations of potential evaporation. Unfortunately, the temperature and specific humidity at the reference height of two meters needed for the Penman calculation were not measured. These variables were approximated by temperature and humidity data available at a height of ~1.2 m. In the daytime, use of the 1.2 m temperatures would overestimate the saturation vapor pressure at 2 m, whereas the 1.2 m specific humidity would overestimate the 2 m specific humidity. Therefore, the net error in the estimated 2 m humidity deficit is smaller than the errors in the estimated 2 m temperature and specific humidity.

Since the surface temperature was not measured and cannot be simply defined over land surfaces, the

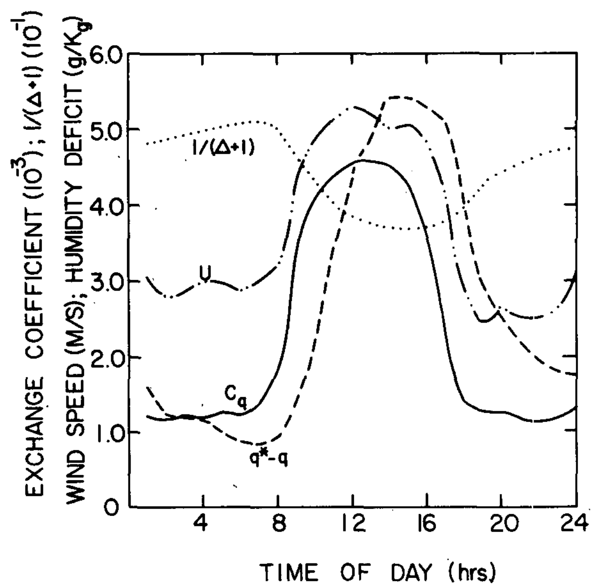

FIG. 1b. Mean diurnal variation of wind speed (dot-dash), specific humidity deficit (broken), Louis stability-dependent exchange coefficient C_q (solid), and temperature-dependent coefficient $1/(1 + \Delta)$ (dotted).

TABLE 1. Limiting cases of potential evaporation for (a) the original Penman relationship and (b) the modified Penman relation.

	Surface energy balance							
	Radiation term	Aerodynamic term				Latent heat flux		
Case	$R_n - S$	u	C_q	$q^* - q$	Net contribution	E	Sensible heat flux	Type of evaporation
				(a) original Penman relationship				
1a	0	0	>0	0	0	0	0	None
2a	<0	0	>0	0	0	<0	<0	Dewfall
3a	>0	0	>0	0	0	>0	>0	Free convection
4a	0	>0	>0	0	0	0	0	None
5a	<0	>0	>0	0	0	<0	<0	Dewfall
6a	>0	>0	>0	0	0	>0	>0	
7a	0	0	>0	>0	>0	>0	<0	(See text)
8a	<0	0	>0	>0	>0	(1)	<0	(See text)*
9a	>0	0	>0	>0	>0	>0	>0	Free convection
10a	0	>0	>0	>0	>0	>0	<0	Mechanical turbulence
11a	<0	>0	>0	>0	>0	(1)	<0	*
12a	>0	>0	>0	>0	>0	>0	<0	†
13a	>0	>0	>0	>0	>0	>0	0	†
14a	>0	>0	>0	>0	>0	>0	>0	†
				(b) modified Penman relationship				
1b	0	0	0	0	0	0	0	None
2b	<0	0	0	0	0	<0	<0	Dewfall
3b	>0	0	>0	0	0	>0	>0	Free convection
4b	0	>0	>0	0	0	0	0	None
5b	<0	>0	>0	0	0	<0	<0	Dewfall
6b	>0	>0	>0	0	0	>0	>0	
7b	0	0	0	>0	0	0	0	None
8b	<0	0	0	>0	0	<0	<0	Dewfall
9b	>0	0	>0	>0	>0	>0	>0	Free convection
10b	0	>0	>0	>0	>0	>0	<0	Mechanical turbulence
11b	<0	>0	>0	>0	>0	(1)	<0	*
12b	>0	>0	>0	>0	>0	>0	<0	Mechanical turbulence
13b	>0	>0	>0	>0	>0	>0	0	Mechanical turbulence
14b	>0	>0	>0	>0	>0	>0	>0	

* Depends on magnitude of each contribution.
† Because the original relationship is independent of stability, 12a = 13a = 14a.

surface-based bulk Richardson number could not be computed. Instead, the layer Richardson number is computed using observations at the 1 and 4 m levels. Because the exchange coefficient is a slowly varying function of the Richardson number, except near neutral stability, the error in the estimation of the surface-based bulk Richardson number will normally cause much smaller errors in the exchange coefficient. Note that we cannot re-integrate the similarity theory between 1 and 4 m to obtain a new exchange coefficient relationship because the Penman relationship demands integration from the surface; i.e., the bulk aerodynamic relationship for moisture must be defined with respect to surface properties so that saturation can be assumed, allowing surface vapor pressure to be related to surface temperature. We assess the importance of these potential errors in Section 8 where the exchange coefficients are also computed using an iterative procedure.

To compute the typical diurnal variation, parameters for each hour are averaged over all of the Wangara days, including both sunny and cloudy days. Since ρ and L_v vary by only a small percentage during the day, they are considered constant and set equal to 1.275 kg m^{-3} and 2.5×10^6 J kg^{-1}, respectively. The roughness length is assigned to be 1.2 mm (Clarke *et al.*, 1971). Figs. 1–2 show the diurnal variation of the remaining variables averaged over 40 days. Occasional missing observations contribute to some of the hour-to-hour noise. For most hours, less than 10% of the observations of a given variable were missing. As expected, the stability-dependent exchange coefficient increases in the morning to a maximum value occurring in the early afternoon, dropping off rapidly later in the afternoon to an almost time-independent nocturnal value. The inferred Penman exchange coefficient $C_{qp} = f(u)/u$, where $f(u)$ is the original Penman wind function, reaches a minimum in the afternoon, violating physical expectations.

Values are also averaged for days on which significant instability (Obukhov length < 10 m) occurred in the afternoon. Nine such days are found. On these days, the afternoon exchange coefficient exceeds that inferred from the Penman relationship by almost a factor of 2 (Fig. 2).

The diurnal variation of wind function $f(u) = C_q u$ corresponding to the stability-dependent aerodynamic expression exhibits significantly greater diurnal variation than the wind function of the unmodified Penman relationship even when averaged over all days (Fig. 3). Here the Penman wind function follows a diurnal pattern close to that corresponding to that wind function with a constant neutral value of the exchange coefficient, but with a smaller decrease at night.

Figure 4 shows the diurnal variation of the radiation term and the various aerodynamic terms. The radiation expression peaks around noon, whereas the aerodynamic expressions peak in early afternoon. The aerodynamic terms are as large or nearly as large as the radiation term in contrast to some unstable summertime cases where the radiation term is significantly larger than the aerodynamic term.

As expected, the aerodynamic term using the stability-dependent exchange coefficient C_q exhibits, on the average, considerably more diurnal variation than

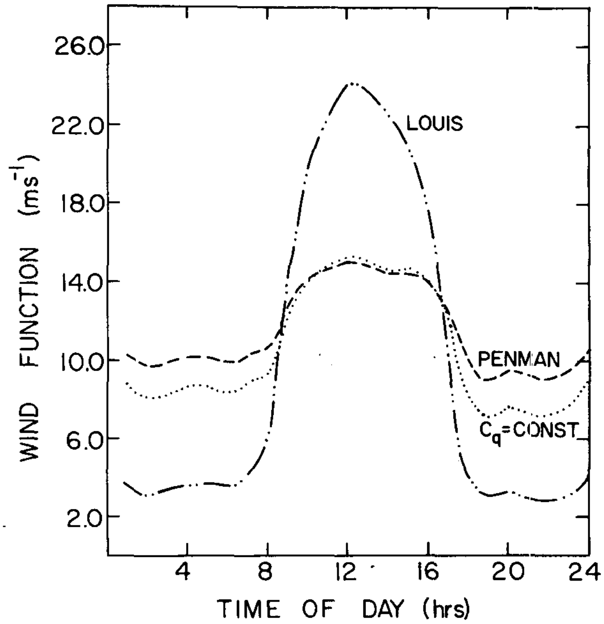

FIG. 3. The averaged diurnal variation of the wind function as computed from the Louis stability-dependent exchange coefficient (dot-dash), the original Penman formulation (broken) and constant exchange coefficient (dotted).

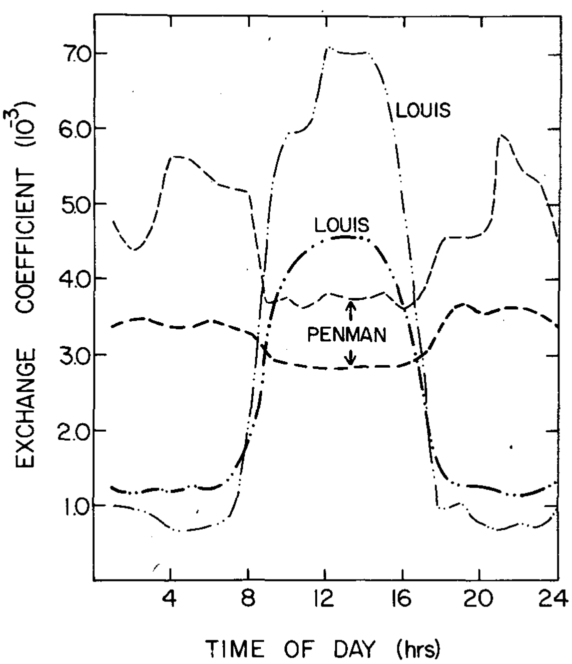

FIG. 2. Averaged diurnal variation of the exchange coefficient as computed from the Louis formulation (thick dot-dash), and as inferred from the original Penman wind function $f(u)/u$ (thick broken). Also shown are averages for the nine days with significant afternoon instability (thin dot-dash and thin broken).

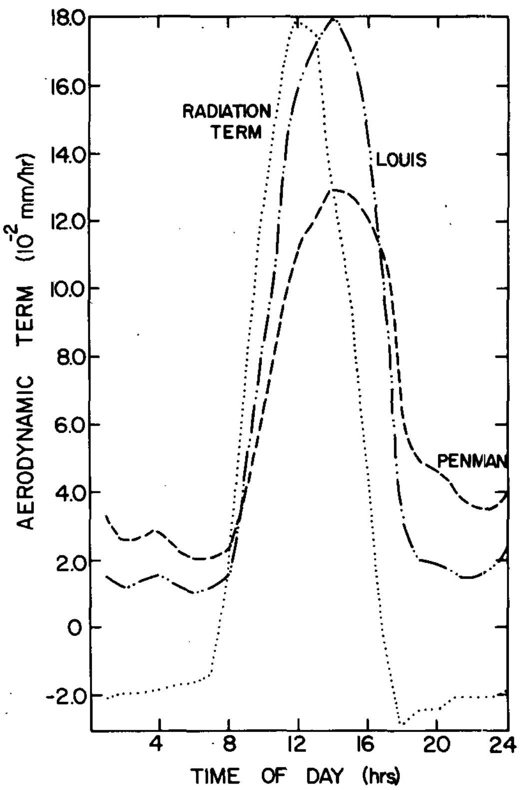

FIG. 4. The averaged diurnal variation of the aerodynamic term as computed from the Louis stability-dependent exchange coefficient (dot-dash), the unmodified Penman aerodynamic term (broken) and the radiation term (dotted).

the aerodynamic term of the original Penman relationship. The aerodynamic term of the original Penman relationship is similar to the aerodynamic term with a constant neutral exchange coefficient. Averaged over all days, the stability-dependent Penman relationship predicts the afternoon aerodynamic term to be about 40% greater than that of the original Penman relationship in the afternoon and about 50% less than that of the original Penman relationship during the night.

6. Nonlinear diurnal dependence

In most applications of the Penman relationship, the daily total evaporation is estimated by using daily averages of wind speed, net radiation, humidity deficit and temperature, and by neglecting the influence of atmospheric stability. Such calculations will incur errors due to neglect of stability influences and due to the nonlinear interaction (correlation) between the diurnal variation of variables which appear as products in the Penman relationship. For example, Jobson (1972) found that nonlinear interaction between the diurnal variation of vapor pressure and wind speed could occasionally lead to significant errors with use of daily-averaged variables in Dalton's relationship. However, on 90% of the days, such an error was found to be less than 10%. This error remains generally less than 25% even when monthly-averaged variables are used (Hage, 1975).

The results of Doorenbos and Pruitt (1977), also discussed by Stigter (1980), indicate that use of the Penman relation for predicting 24 h evapotranspiration from a well-watered grass reference crop could incur errors of 50% or more. The Penman relationship was found usually to underestimate water loss with strong wind speed and weak radiative heating. This underestimation decreased or reversed to an overestimation with weak wind speeds and strong surface radiative heating. Although stomatal resistance may have been a factor, the above variation of errors is consistent with the influence of atmospheric stability on potential evaporation through the stability-dependent exchange coefficient and its correlation with other variables in the aerodynamic term.

We now consider the difference between calculating daily potential evaporation from daily-averaged variables versus summing potential evaporation values computed from hourly variables. We refer to the first case as "linearized potential evaporation" and the second as "integrated potential evaporation."

Table 2 summarizes the differences between different expressions averaged over the 40 days of the Wangara experiment. We first note that the linearized radiation term averages 13% less than the integrated radiation term. This underestimation is due to correlation between the diurnal variation of the radiation (less soil heat flux) and the temperature-dependent coefficient of the radiation term.

TABLE 2. The daily aerodynamic term averaged over the 40 Wangara days (mm).

Method	Averaged value	Difference from integrated modified (%)	Averaged absolute difference (%)
Linearized Penman	1.35	−4	15
Integrated Penman	1.42	+1	12
Linearized, stability dependent	0.89	−36	36
Integrated, stability dependent	1.40	—	—

The linearized and daily-integrated versions of the aerodynamic terms of the original Penman relationship average within 5% of each other. The average absolute difference between the two versions is only 6%. Even the maximum difference of the 40 individual days is only 11%. This indicates that the linearization due to use of daily-averaged values does not lead to significant errors in the original Penman relationship.

However, the linearization of the stability-dependent aerodynamic term does create significant errors. Here the linearized aerodynamic term averages less than 60% of the integrated aerodynamic term. Differences on some individual days with large variations in stability exceeded 90% of the integrated term. It is concluded that use of the daily-averaged exchange coefficient causes large underestimation of the daily potential evaporation as explicitly shown in Section 7. We also found that use of a neutral value of the exchange coefficient, corresponding to a logarithmic wind profile, underestimates the daily evaporation as previously concluded by Stricker and Brutsaert (1978).

The aerodynamic term of the integrated original Penman averages, perhaps fortuitously, within 1% of that of the integrated stability-dependent Penman with an absolute difference averaging only 12%. A maximum individual daily difference of only 24% of the integrated modified term is found on a day with a large diurnal variation in stability.

The daily values of the original Penman relationship agree rather closely with the more complete version, partly due to cancellation of underestimation in the daytime and overestimation at night. This agreement can also be attributed to the fact that the estimated roughness of 1.2 mm is close to the 1.37 mm thought to be representative of pan conditions used to calibrate the original Penman relationship (Thom and Oliver, 1977). However, similarity of roughness lengths may be a lesser factor since the potential evaporation does not seem to be especially sensitive to the roughness length (Thom and Oliver, 1977).

Similarly, the linearized aerodynamic term of the original Penman equation approximates the aerodynamic term of the integrated, stability-dependent Penman relationship reasonably well. The difference between them averages only 4% and the absolute difference averages only 15%. The maximum individual difference, about 28% of the integrated term, also occurs on a day with a large diurnal variation in stability. We conclude that for the data considered here, the original Penman relationship has been effectively calibrated to predict the daily total evaporation even though it performs poorly on an hourly basis. We further conclude that when only 24 h averages are available, the original Penman is preferable to the stability-dependent Penman, at least without further calibration.

7. Interactive terms

To study the source of errors due to use of daily-averaged variables, each variable has been partitioned into a daily mean denoted by an overbar and an hourly deviation from the daily mean denoted by a prime. Diurnal variations of density and specific latent heat are much smaller than diurnal variations of other parameters and are therefore assigned to be constant with values of $\bar\rho = 1.275$ kg m^{-3} and $\bar L_v = 2.5 \times 10^6$ J kg^{-1}. Considering the coefficients $1/(\Delta + 1)$ and $\Delta/(\Delta + 1)$ each to be a single variable and substituting the partitioned variables into the Penman relationship, we obtain

$$\bar E = \bar E_R + \overline{\rho L_v}\Bigg[\overset{(1)}{\overline{\frac{1}{\Delta+1}}\,\overline{(q^*-q)}(a+b\bar u)} + \overset{(2)}{b\,\overline{\frac{1}{\Delta+1}}\,\overline{u'(q^*-q)'}} + \overset{(3)}{b\overline{(q^*-q)}\,\overline{\left(\frac{1}{\Delta+1}\right)'u'}} + \overset{(4)}{(a+b\bar u)\overline{\left(\frac{1}{\Delta+1}\right)'(q^*-q)'}} + \overset{(5)}{b\overline{\left(\frac{1}{\Delta+1}\right)'u'(q^*-q)'}}\Bigg] \quad (13)$$

for the original Penman relationship, where $a = 3.9 \times 10^{-3}$ and $b = 2.1 \times 10^{-3}$; for the stability-dependent Penman

$$\begin{aligned}\bar E_a = \bar E_R + \overline{\rho L_v}\Bigg[&\overset{(1)}{\overline{\frac{1}{\Delta+1}}\,\bar C_q\bar u\overline{(q^*-q)}} + \overset{(2)}{\overline{\frac{1}{\Delta+1}}\,\bar C_q\overline{u'(q^*-q)'}} + \overset{(3)}{\overline{\frac{1}{\Delta+1}}\,\bar u\overline{C'_q(q^*-q)'}} + \overset{(4)}{\overline{\frac{1}{\Delta+1}}\,\overline{(q^*-q)}\,\overline{C'_q u'}} \\ &\times \overset{(5)}{\overline{\frac{1}{\Delta+1}}\,\overline{C'_q u'(q^*-q)'}} + \overset{(6)}{\bar C_q\bar u\overline{\left(\frac{1}{\Delta+1}\right)'(q^*-q)'}} + \overset{(7)}{\bar C_q\overline{(q^*-q)}\,\overline{\left(\frac{1}{\Delta+1}\right)'u'}} + \overset{(8)}{\bar C_q\overline{\left(\frac{1}{\Delta+1}\right)'u'(q^*-q)'}} \\ &+ \overset{(9)}{\bar u\overline{(q^*-q)}\,\overline{\left(\frac{1}{\Delta+1}\right)'C'_q}} + \overset{(10)}{\bar u\overline{\left(\frac{1}{\Delta+1}\right)'C'_q(q^*-q)'}} + \overset{(11)}{\overline{(q^*-q)}\,\overline{\left(\frac{1}{\Delta+1}\right)'C'_q u'}} + \overset{(12)}{\overline{\left(\frac{1}{\Delta+1}\right)'C'_q u'(q^*-q)'}}\Bigg], \quad (14)\end{aligned}$$

where

$$\bar E_R = \left[\overset{(1)}{\overline{\frac{\Delta}{\Delta+1}}\,\overline{(R_n - S)}} + \overset{(2)}{\overline{\left(\frac{\Delta}{\Delta+1}\right)'(R_n - S)'}}\right]. \quad (15)$$

Table 3 summarizes the magnitude of the various terms averaged over the 40 Wangara days.

The nonlinear interaction term in the radiation expression averages about 13% of the linear term averaged over all 40 days; it may exceed 25% of the linear term on days with large diurnal variation of temperature and net radiation (less soil heat flux).

In the original Penman aerodynamic expression, the nonlinear interaction between wind and specific humidity deficit (term 2) and between specific humidity deficit and the temperature-dependent coefficient (term 4) are found to be the most significant of the nonlinear terms, approximately 9% and −8% of the linear term, respectively. On a day with large diurnal variation of atmospheric variables, these nonlinear terms reach about 20% of the magnitude of the linear term. They result from higher wind speed, temperature and specific humidity deficit during the afternoon compared to nocturnal periods. Note that these two nonlinear terms are of opposite sign and approximately cancel. This explains why use of the daily-averaged values in the original Penman did not cause significant errors, at least with data analyzed here.

Of the eleven nonlinear terms in the stability-dependent aerodynamic term, seven are found to be relatively unimportant, (terms 5 and 7–12), summing to

TABLE 3. Daily summed nonlinear terms averaged over the 40 Wangara days (mm).

Term	Averaged value	Ratio to linear term
	Original aerodynamic term	
(1)	1.45	1.00
(2)	0.13	0.09
(3)	−0.02	−0.02
(4)	−0.11	−0.08
	Modified aerodynamic term	
(1)	1.03	1.00
(2)	0.14	0.14
(3)	0.23	0.29
(4)	0.18	0.19
(5)	0.03	0.04
(6)	−0.08	−0.08
(7)	−0.03	<0.01
(8)	−0.01	−0.01
(9)	−0.05	−0.05
(10)	−0.01	−0.02
(11)	−0.01	−0.01
(12)	−0.02	−0.02

less than −9% of the linear term. The correlation between the exchange coefficient (C_q) and specific humidity deficit ($q^* - q$), and the correlation between the exchange coefficient (C_q) and wind (u) lead to the most important nonlinear terms which average 29% and 19% of the linear term, respectively. On a day with particularly high diurnal variation of atmospheric variables, these two nonlinear terms are found to be 57% and 26% of the linear term, respectively. This strong correlation between diurnal variations of the stability-dependent exchange coefficient, wind and humidity accounts for the large errors resulting from the use of daily-averaged variables in the stability-dependent Penman relationship.

8. Iterative results

The Louis formulation is expected to incur errors associated with the approximation of the original similarity theory. Errors also result from the use of the layer Richardson number between 1 and 4 meters in lieu of the Richardson number evaluated between the surface roughness height and the standard level of 2 m. Note that in modeling situations, surface temperature is usually determined from the surface energy balance between radiation, evaporation and heat fluxes into the atmosphere and soil. The radiative temperature associated with this balance is often quite different from the air temperature even if measured down at the roughness height. This is an unavoidable inconsistency in modeling situations whenever a bulk aerodynamic relationship is used in conjunction with a surface energy budget. That is, similarity theories do not apply to the actual surface temperature of the ground even when such temperatures can be defined.

As a check on the modeled stability influence used here, we have compared it with the original similarity expressions of Businger *et al.* (1971). We have integrated the similarity theory between the surface roughness height and 2 m for wind and 1 and 4 m for temperature and then used several iterative approaches cited in the Introduction. The Louis relationships used here seem to lead to a small overestimation of the exchange coefficient in unstable situations during the Wangara experiment, although this disagreement varied somewhat with the choice of iterative scheme. In addition, the original similarity expressions are uncertain in cases of large instability or large stability. Lower limits on the Obukhov length must be imposed for stable situations in order to ensure physically realistic behavior and in some cases ensure convergence of the iterative scheme.

We conclude that the qualitative differences between the original Penman relationship and those modified to include stability dependencies are not critically dependent on the stability formulation, as also concluded by Stricker and Brutsaert (1978). However, even the modified formulations are only approximate and a precise quantitative evaluation of the original Penman relationship is not possible.

9. Radiation Richardson number

The iterative procedure may be too cumbersome for many applications, while the evaluation of the Richardson number requires temperature at two levels, not typically available in modeling and routine observational situations. The Richardson number can be modified by replacing the temperature difference with a dependence on radiation. The resulting parameter is more "external" than the usual Richardson number, since the temperature difference and turbulent fluxes are directly coupled. The surface heat flux is uniquely related to net radiation under conditions of potential evaporation and negligible heat flux to the soil. However, in general, the evaporation is less than potential and the ratio of the heat flux to the surface net radiation varies.

A "radiation Richardson" number can be developed by beginning with the flux Richardson number

$$R_f \equiv \frac{\frac{g}{\theta}\overline{w'\theta'}}{\frac{\partial \bar{\mathbf{v}}}{\partial z}\overline{w'\mathbf{v}'}}, \tag{16}$$

where $\overline{w'\theta'}$ is the surface temperature flux, $\overline{w'\mathbf{v}'}$ the surface momentum flux and $\bar{\mathbf{v}}$ the mean flow vector. We replace the seldom-measured surface heat flux with the radiative temperature flux less soil heat flux

$$R = \frac{(R_n - S)}{\rho c_p}. \tag{17}$$

Here R_n is positive with net downward radiative flux and is expressed in W m^{-2} in which case R is in units of K m s^{-1}. In most practical situations, S would be neglected. Scaling velocity fluctuations with u, the wind speed at height z, a scale value for the flux Richardson number (16) becomes proportional to the radiation Richardson number

$$\mathrm{Ri}_{\mathrm{rad}} \equiv (g/\theta)Rz/u^3. \quad (18)$$

The relationship between the radiation Richardson number and atmospheric stability depends on the actual evaporation rate so that the radiation Richardson number is only a crude estimate of atmospheric stability. However, since the main variation of the exchange coefficient occurs in the transition between stable and unstable cases, the crude estimate of stability based on the radiation Richardson number will be of utility.

To develop the intended use of the radiation Richardson number, we regress z/L on the radiation Richardson number where L, the Obukhov length, is computed iteratively using the similarity relationships of Businger *et al.* (1971). The height z is again 2 m. Cases where net radiative heat gain is positive and z/L is stable and vice versa are eliminated since these cases normally occur in transitional periods when similarity theory is not expected to apply. Fortunately, potential evaporation rates are small during such periods.

The radiational Richardson number and z/L are linearly correlated with a coefficient of 0.95 for the unstable cases and 0.57 for stable cases. However, distributions of these parameters are strongly skewed. The cube root of z/L and the radiation Richardson number are more normally distributed and will be used for the regression relationships. Note that $(z/L)^{1/3}$ is inversely related to the surface friction velocity while $(\mathrm{Ri}_{\mathrm{rad}})^{1/3}$ is inversely proportional to the wind speed, so that the resulting regression relationship is analogous to a resistance law.

The cube root of z/L is correlated to $(\mathrm{Ri}_{\mathrm{rad}})^{1/3}$ with a correlation coefficient of 0.90 in the unstable case and 0.77 in the stable case. The regression relationships for both cases are

$$\left.\begin{aligned} \left(\frac{z}{L}\right)^{1/3} &= -8.64(\mathrm{Ri}_{\mathrm{rad}})^{1/3} - 0.09, \quad \text{stable case} \\ \left(\frac{z}{L}\right)^{1/3} &= -15.29(\mathrm{Ri}_{\mathrm{rad}})^{1/3} - 0.13, \quad \text{unstable case} \end{aligned}\right\}. \quad (19)$$

Both relationships predict that z/L approaches $\sim -10^{-3}$ as the net radiation vanishes. This small constant has no special significance for the near neutral case but rather improves the fit over the range of the values of the radiation Richardson number. A higher order model is not justified because of the very approximate nature of this development.

The foregoing regression relationships (19) were used to estimate z/L and subsequently to compute the exchange coefficients for the Wangara data. These exchange coefficients averaged over the forty days appear to lead to an underestimation of the stability influence (Figs. 5 and 6). With wetter surface conditions, this technique may overestimate the stability influence. However, these simple explicit relationships based on the radiation Richardson number should be a significant improvement upon complete neglect of the influence of atmospheric stability and at the same time do not require additional observations as with other stability parameters.

10. Penman–Monteith relationship

To include the influence of stomatal control, the Penman relationship is often multiplied by a plant coefficient which is generally less than unity. As an alternative, the Penman–Monteith relationship (Monteith, 1965) is frequently used. This approach introduces the influence of vegetation properties through a surface resistance factor r_s and the aerodynamic exchange coefficient is absorbed into a coefficient for aerodynamic resistance to atmospheric vapor flux r_a through the relationship

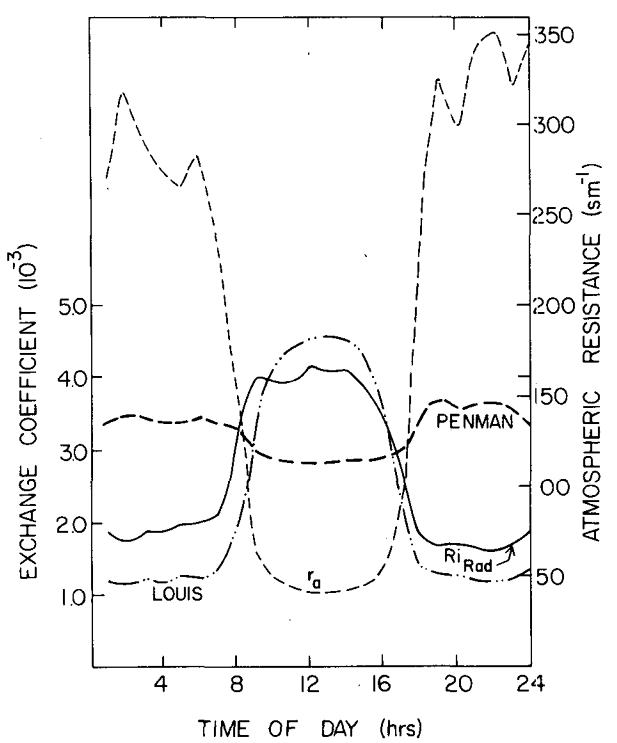

FIG. 5. The averaged diurnal variation of the exchange coefficient based on the Louis formulation (dot-dash), the radiation Richardson number-regression relationship (solid) and as inferred from the Penman wind function (broken). Also shown is the atmospheric resistance coefficient (thin-broken).

$$r_a = \frac{1}{(C_q u)}, \tag{20}$$

where C_q is parameterized according to (11). Thus the resistance coefficient is sensitive to atmospheric stability. We have computed the diurnal variation of the resistance coefficient from the exchange coefficient averaged over all the days for each hour. The resistance coefficient cannot be averaged directly since during very stable conditions, the resistance coefficient becomes orders of magnitude greater than typical values and theoretically can become infinite.

Diurnal variations of r_a, as computed from the Wangara data, are plotted in Fig. 5. Since the diurnal variation of r_a is substantial, the neglect of stability and nonlinear interactions in the Penman–Monteith expression is expected to lead to large errors. The diurnal variation of r_a could, in turn, be used to assess the diurnal variation of the coefficient α in the Priestly–Taylor model by employing the model of de Bruin (1983).

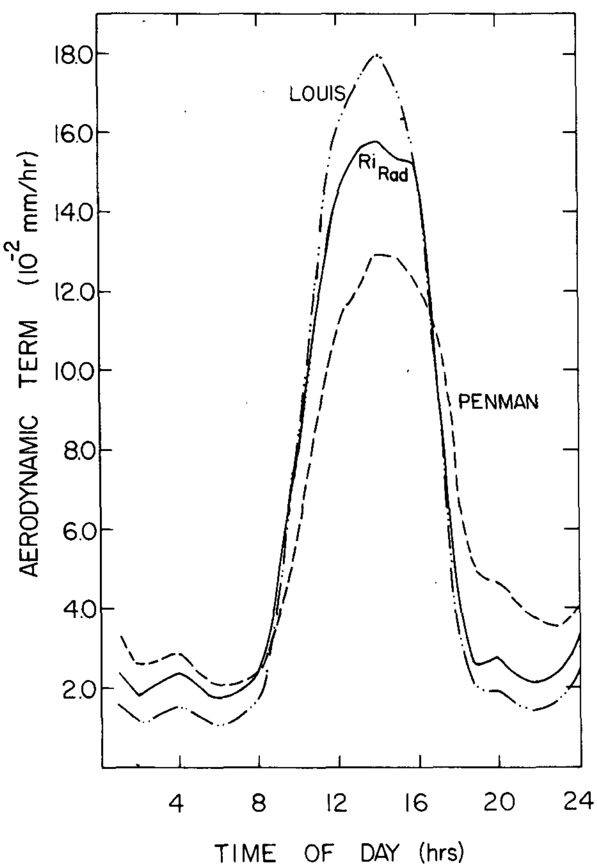

FIG. 6. The averaged diurnal variation of the aerodynamic term computed from the exchange coefficient based on the Louis formulation (dot-dash), the radiation Richardson number-regression relationship (solid) and the unmodified Penman aerodynamic term (broken).

Relationship (20) along with either (11), (19) or the procedure in Section 8 would allow inclusion of the stability influence in the Penman–Monteith expression.

11. Conclusions

The potential evaporation, as related exclusively to atmospheric variables, is found to be quite sensitive to the diurnal variation of the exchange coefficient appearing in the bulk aerodynamic relationship (Dalton's law). The usual neglect of such diurnal variations of stability leads to a factor of 2 smaller estimates of the values for the aerodynamic term on afternoons with modest instabilities, although the differences are reduced substantially when averaged over all of the days in the Wangara experiment (Section 5). Strong instability (very small Obukhov length) did not occur with the data analyzed here, due to the relatively low winter sun angle and generally significant airflow. In many summer midlatitude situations, the diurnal variation of stability will be greater than reported here. Over a fully vegetated surface with moist soil and significant airflow, the diurnal variation will often be less than found here.

Although responding inadequately to diurnal variations, the original Penman relationship predicted daily totals of the potential evaporation which are in good agreement with values predicted by the more complete relationship. This can be attributed to partial cancellation of the daytime and nocturnal influences of atmospheric stability and the similarity between roughness length in the Wangara experiment and that corresponding to the original calibration of the Penman relationship. Use of daily-averaged variables in lieu of summing hourly estimates from the original Penman relationship led to little error.

We have also developed a new formulation for computing the stability-dependent exchange coefficient by defining a "radiation Richardson number" (Section 9). Although less accurate, this simple formulation uses only variables that are already required for evaluation of the unmodified Penman relationship.

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