Phys 163; HW #10 Due Thurs. 11/16 (Midterm on Tues 11/21)

NO LATE HOMEWORK ACCEPTED DUE TO MIDTERM!!

1. Suppose a wavefunction of an electron in a Hydrogen atom is given by

$$\Psi = A\left[\left|\ell=1, m_l=-1, m_s=1/2\right\rangle + 2i\left|\ell=2, m_l=-1, m_s=\frac{1}{2}\right\rangle - 2\left|\ell=1, m_l=0, m_s=-\frac{1}{2}\right\rangle\right].$$

(a) Normalize the wavefunction (find A).
(b) If you measure L^2, what do you find, and with what probabilities?
(c) If you measure Lz, what do you find, and with what probabilities?
(**d**) Use the Clebsch-Gordon table to rewrite this state in the $|l,j,m_j\rangle$ basis. Combine like terms if (and only if!) all three of those numbers are identical.
(e) What values might result from a J^2 measurement (total angular momentum squared), and what are the probabilities of each possible result? (Give values, not quantum numbers!)

2) A two-particle spin state (both spin-1/2) can always be written as $a\uparrow\uparrow+b\uparrow\downarrow+c\downarrow\uparrow+d\downarrow\downarrow$. The coefficients a,b,c,d are all complex; assume they are normalized already. What is the probability that the combined angular momentum magnitude of the whole system will be measured to be zero? (Hint: Just use the Born rule!)

3) An electron is in a magnetic field of strength B, where B points in the NEGATIVE z-direction.
Construct a normalized spin state $\begin{pmatrix} a \\ b \end{pmatrix}$ with an energy expectation value of $\frac{eB\hbar}{6m}$.
Hint: Set this up in terms of two equations and two unknowns; then solve!

4) (Use the information on bottom of the next page to make this problem solvable.) A spin-1 particle with gyromagnetic ratio γ is in a magnetic field of strength B; the magnetic field points in the negative z-direction.

A) At time t=0, an experimenter measures the spin angular momentum in the y-direction and gets a result of $-\hbar$. What is the wavefunction at a time t?

B) At time "t" the experimenter immediately measures the spin angular momentum in the x-direction. What values might she get, and what are the probabilities of each?

5. Two electrons in a singlet state (total spin =0) are prepared. Electron #1 is sent to Alice, while electron #2 is sent to Bob.

Alice measures S_z for her electron. Bob measures S_θ, where θ is at the angle θ from the z axis in the x-z plane. (ϕ=0). (Use the results from problem 4.30 to figure out Bob's measured eigenstates.)

A) Find the 4 eigenstates of the *whole system* that correspond to all 4 possible measurement results. (Alice can get one of two results, Bob can get one of two results, so there are 4 total results in all.) Hint: tensor product each of Alice's eigenstates with each of Bob's eigenstates.

B) Find the probabilities of each of the four outcomes if $\theta = 45^\circ$. (Use the Born rule!)

C) Find the probabilities of each of the four outcomes if $\theta = 135^\circ$.

D) For each of the angles in parts B) and C), calculate the expectation value of the product of Alice's result and Bob's result.

Spin Matrices and Normalized Eigenvectors

3D Hilbert Space, in basis defined by a diagonal S_z (highest eigenvalue on top).

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix} : \quad |+\hbar\rangle_x = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \quad |0\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad |-\hbar\rangle_x = \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i\sqrt{2} & 0 \\ i\sqrt{2} & 0 & -i\sqrt{2} \\ 0 & i\sqrt{2} & 0 \end{pmatrix} : \quad |+\hbar\rangle_y = \frac{1}{2}\begin{pmatrix} 1 \\ i\sqrt{2} \\ -1 \end{pmatrix} \quad |0\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad |-\hbar\rangle_y = \frac{1}{2}\begin{pmatrix} 1 \\ -i\sqrt{2} \\ -1 \end{pmatrix}$$