

3.15

$$\vec{B} = (\hat{r}r \cos \phi + \hat{\phi} \sin \phi)$$

$$\oint \vec{B} \cdot d\vec{l} = ?$$

$$\int_{\phi=\pi/2}^0 \sin \phi r d\phi = -1$$

$$\int_{r=1}^2 r \cos \phi dr = \frac{3}{2}$$

$$\int_{\phi=1}^{\pi/2} \sin \phi r d\phi = 2$$

$$\int_{r=2}^1 r \cos \phi dr = 0$$

$$\oint \vec{B} \cdot d\vec{l} = -1 + \frac{3}{2} + 2 = \frac{5}{2}$$

$$b) \nabla \times \vec{B} = \frac{1}{r} \begin{vmatrix} \hat{r} & r\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ r \cos \phi & r \sin \phi & 0 \end{vmatrix} = \frac{1}{r} \hat{z} (\sin \phi + r \sin \phi) = \hat{z} \sin \phi \left(1 + \frac{1}{r}\right)$$

$$\int \nabla \times \vec{B} \cdot d\vec{s} = \int_{\phi=0}^{\pi/2} \int_{r=1}^2 \sin \phi \left(1 + \frac{1}{r}\right) r dr d\phi = \frac{5}{2}$$