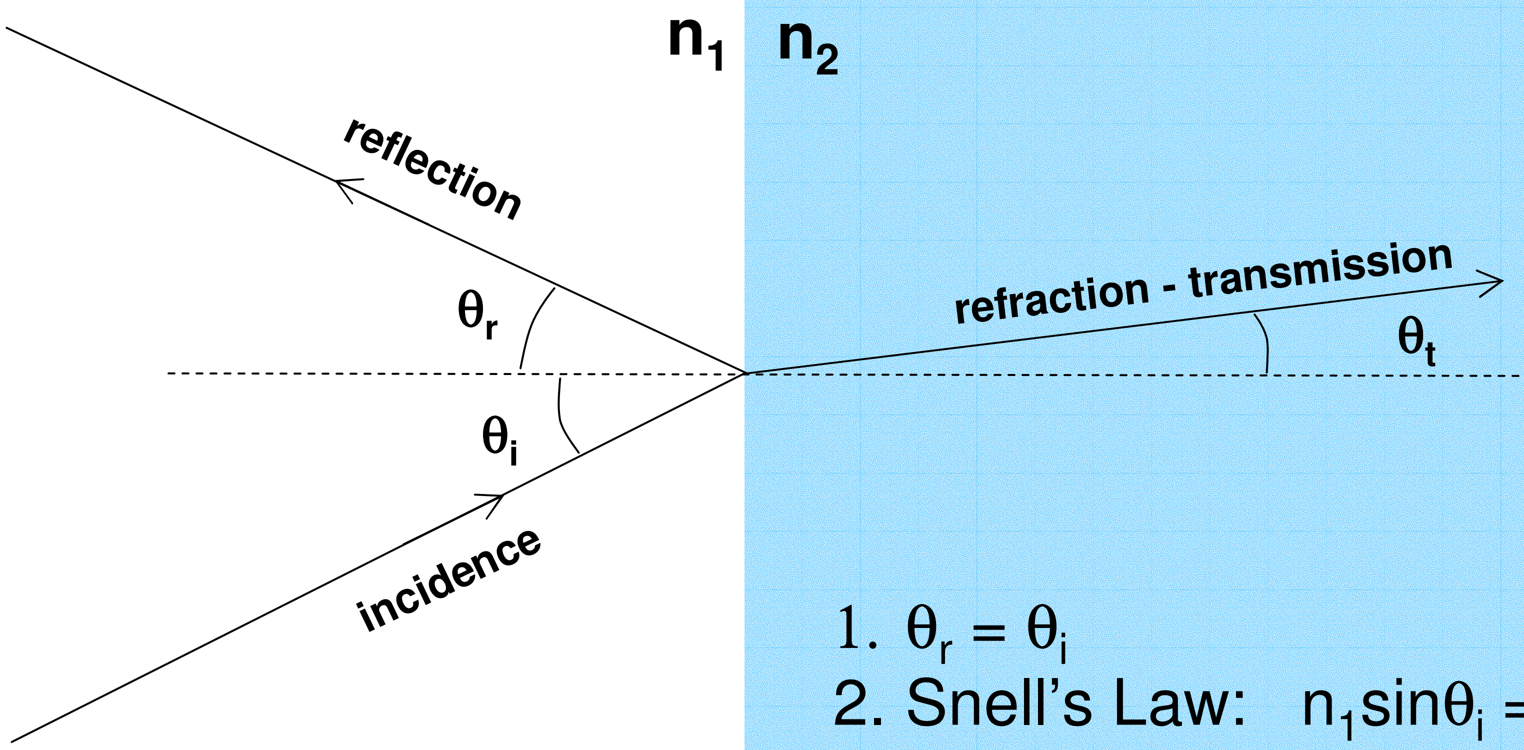
Reflection & Transmission

EE142

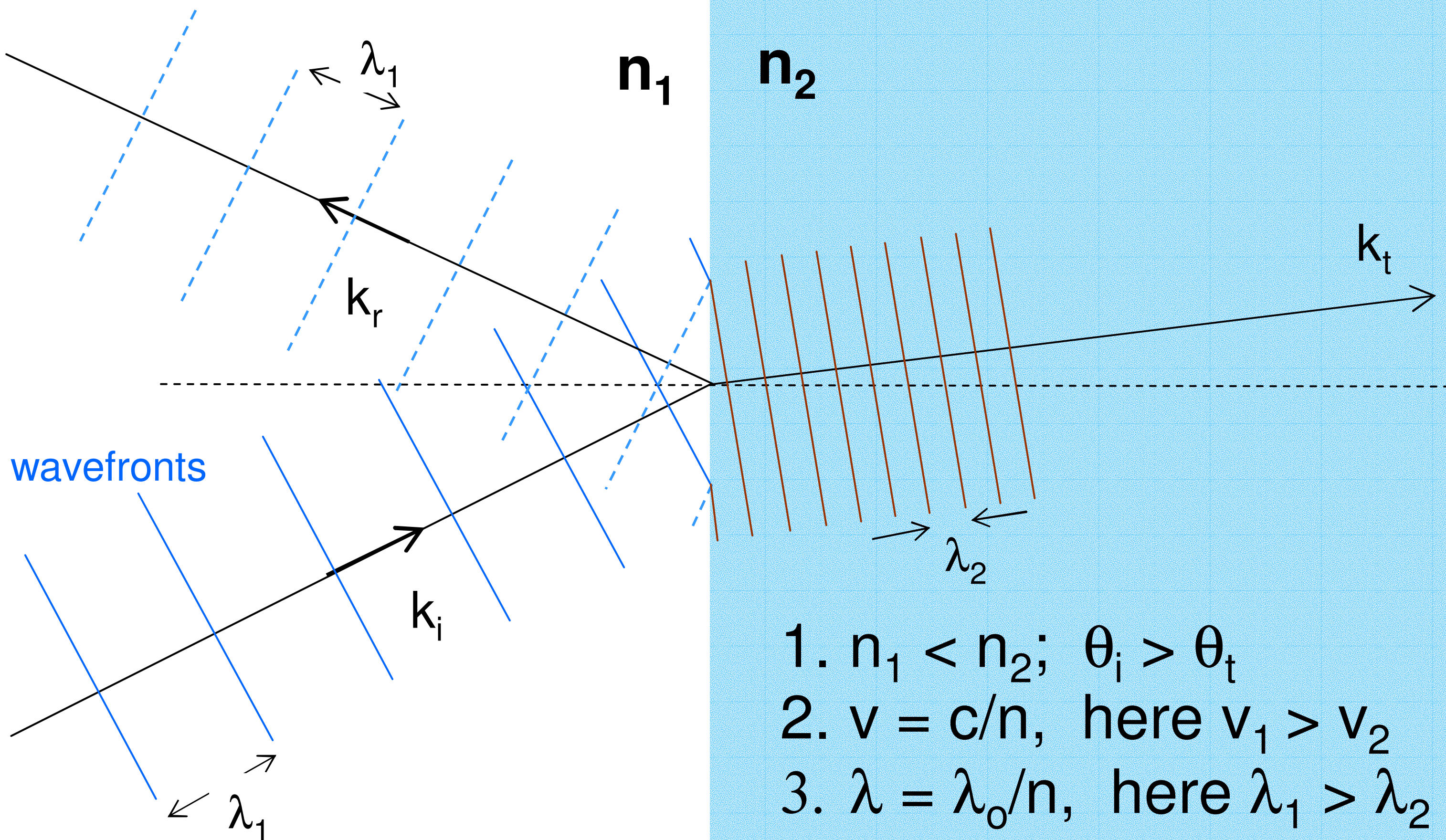
Dr. Ray Kwok

Geometric Optics (EM waves)



1. $\theta_r = \theta_i$
2. Snell's Law: $n_1 \sin\theta_i = n_2 \sin\theta_t$
3. Critical angle: $\sin\theta_c = n_2 / n_1$
4. Total reflection when $\theta_i > \theta_c$ only if $n_1 > n_2$

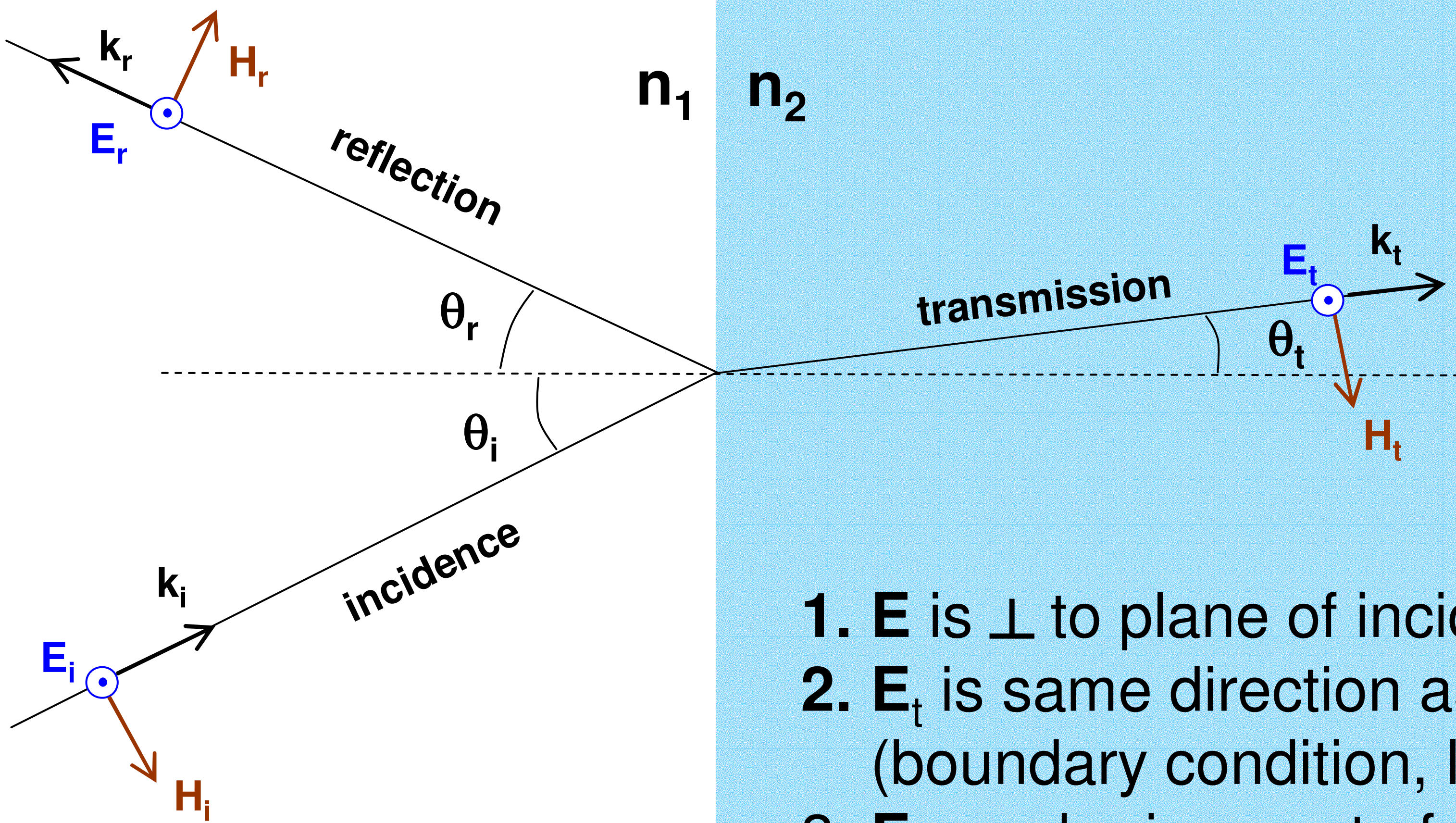
Plane Wave



1. $n_1 < n_2$; $\theta_i > \theta_t$
2. $v = c/n$, here $v_1 > v_2$
3. $\lambda = \lambda_o/n$, here $\lambda_1 > \lambda_2$
4. $f = v/\lambda = c/\lambda_o$ remains the same
   i.e. wave doesn't change color
5. reflection has same λ as incidence

Perpendicular Polarization

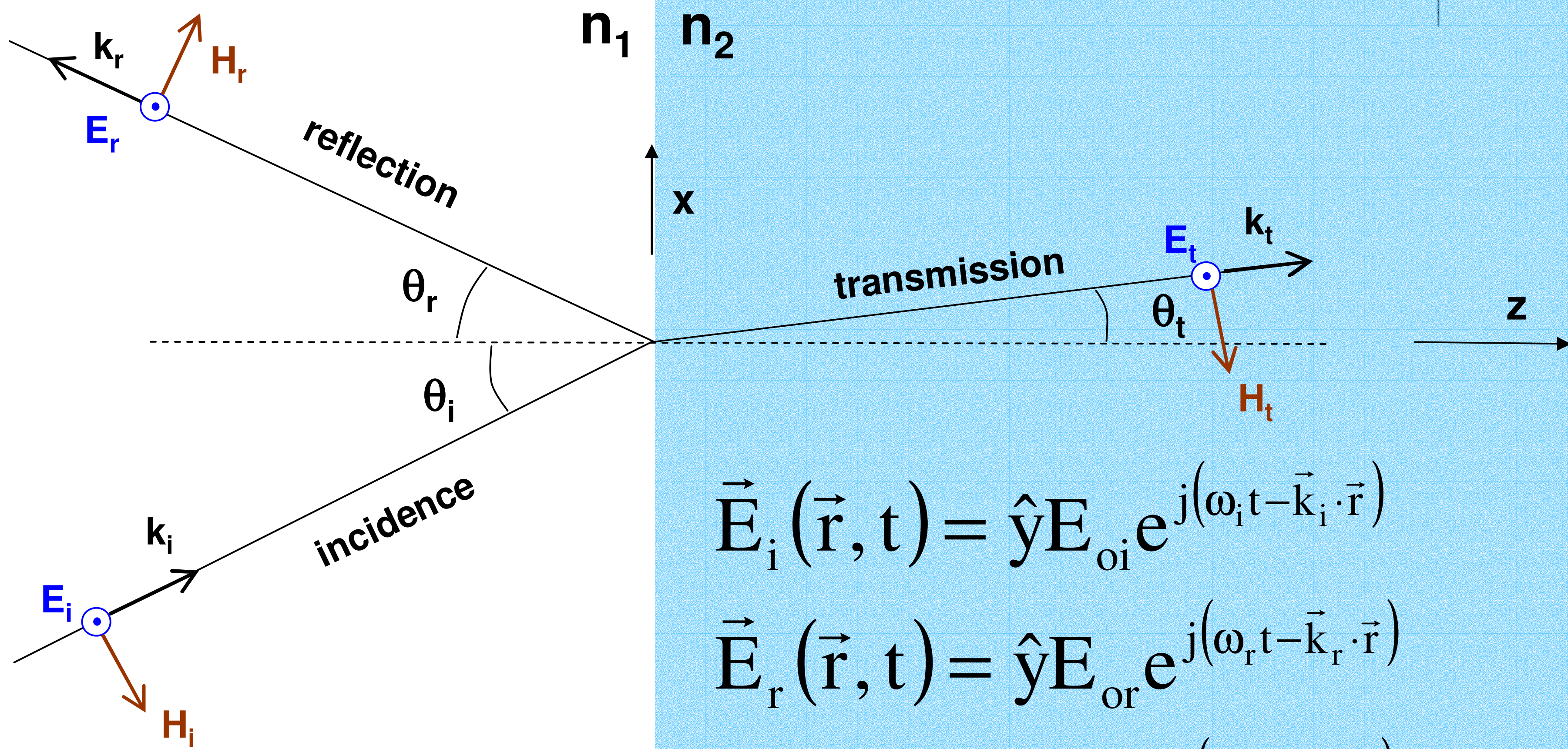
⊥ to "plane of incidence"



1. **E** is ⊥ to plane of incidence
2. $\mathbf{E}_t$ is same direction as $\mathbf{E}_i$ (boundary condition, later)
3. $\mathbf{E}_r$ can be in or out of plane
4. **E** x **H** is along **k**

Wave Equations

⊥ polarization



$$\vec{E}_i(\vec{r},t) = \hat{y}E_{oi}e^{j(\omega_i t - \vec{k}_i\cdot\vec{r})}$$

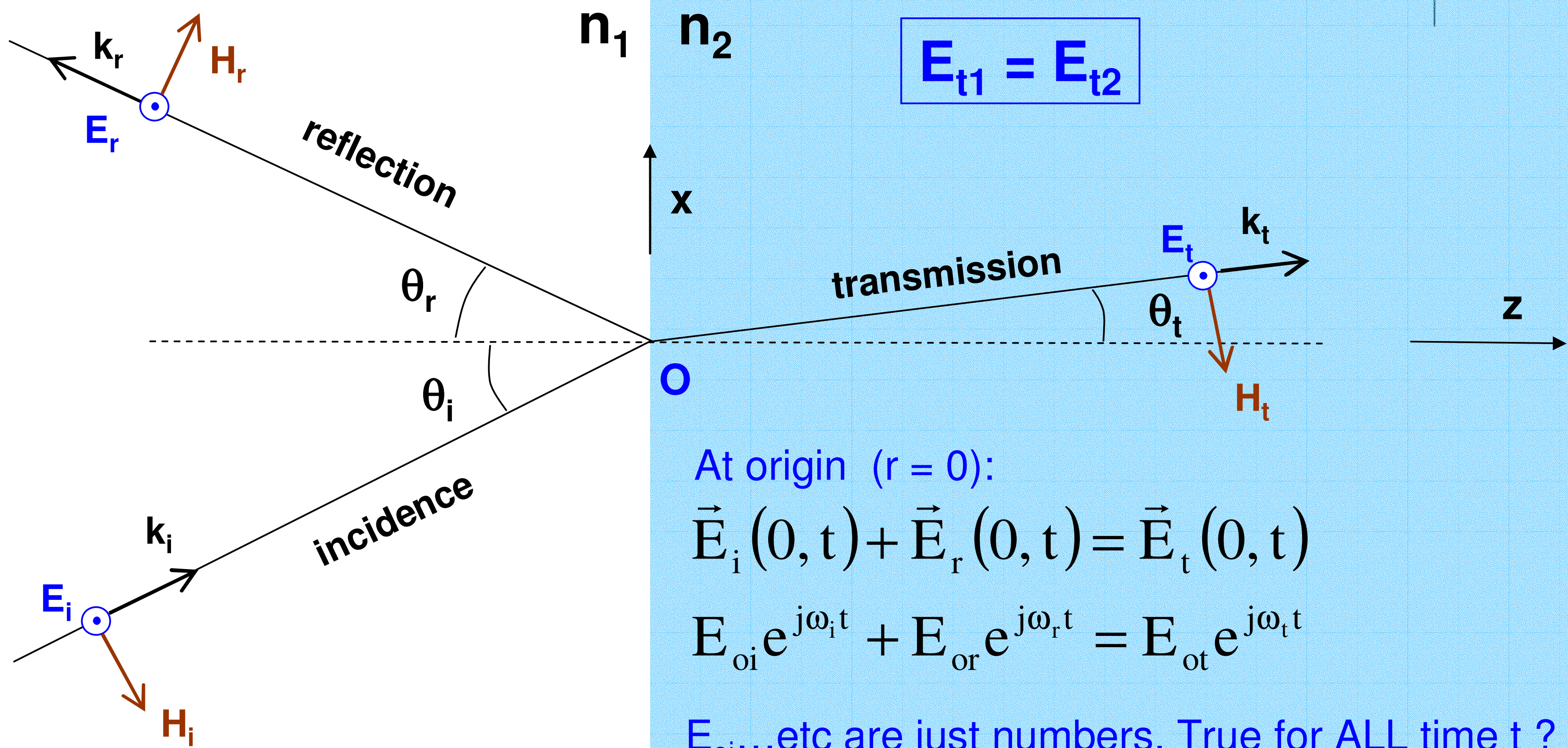
$$\vec{E}_r(\vec{r},t) = \hat{y}E_{or}e^{j(\omega_r t - \vec{k}_r\cdot\vec{r})}$$

$$\vec{E}_t(\vec{r},t) = \hat{y}E_{ot}e^{j(\omega_t t - \vec{k}_t\cdot\vec{r})}$$

What are the corresponding H-fields?

Boundary Condition (1) ⊥ polarization

$$E_{t1} = E_{t2}$$



At origin (r = 0):

$$\vec{E}_i(0,t) + \vec{E}_r(0,t) = \vec{E}_t(0,t)$$

$$E_{oi}e^{j\omega_i t} + E_{or}e^{j\omega_r t} = E_{ot}e^{j\omega_t t}$$

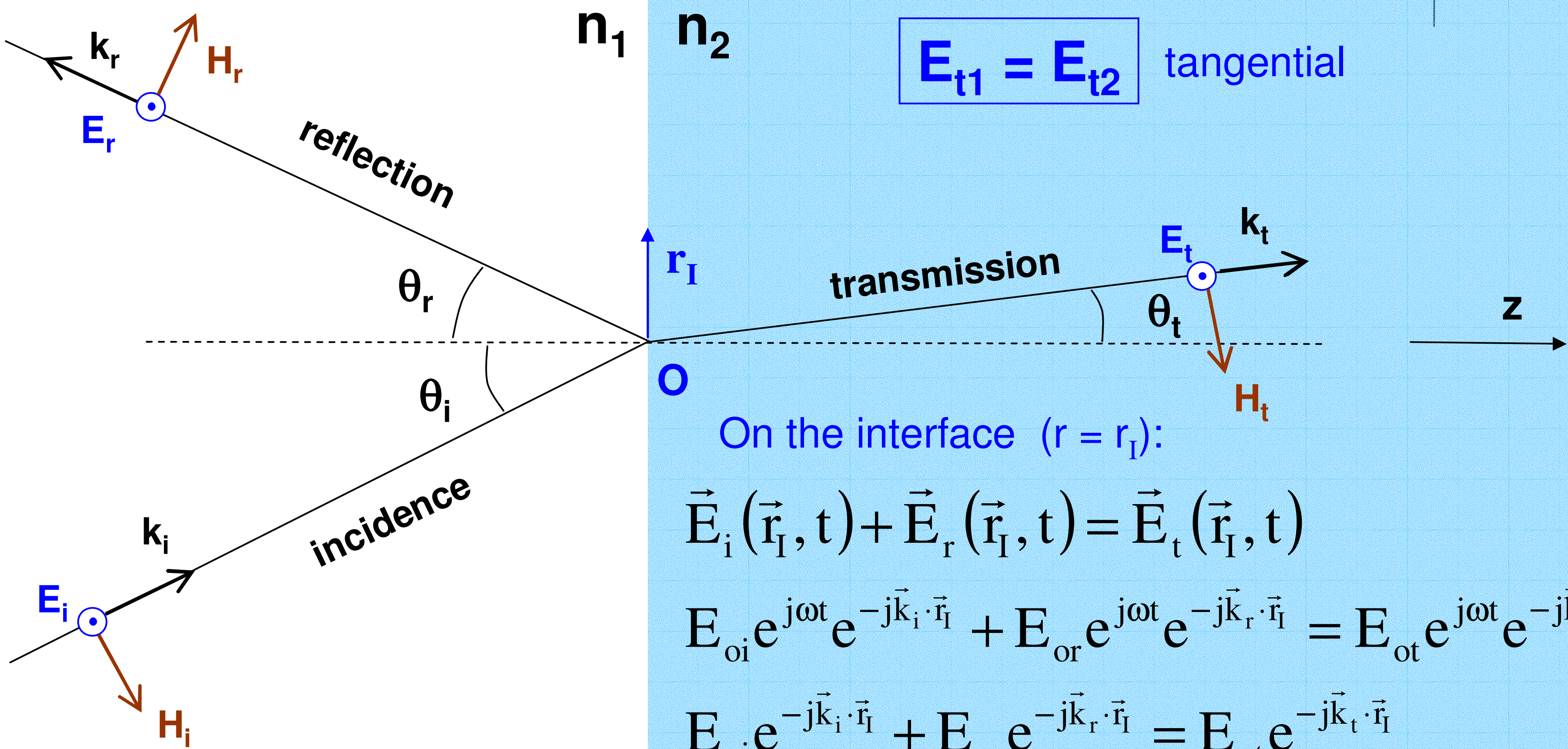
E_{oi}…etc are just numbers. True for ALL time t ?

$$e^{j\omega_i t} = e^{j\omega_r t} = e^{j\omega_t t}$$

$$\omega_i = \omega_r = \omega_t$$

same frequency !!!

Boundary Condition (2) ⊥ polarization



$E_{t1} = E_{t2}$ tangential

On the interface $(r = r_I)$:

$$\vec{E}_i(\vec{r}_I, t) + \vec{E}_r(\vec{r}_I, t) = \vec{E}_t(\vec{r}_I, t)$$

$$E_{oi} e^{j\omega t} e^{-j\vec{k}_i \cdot \vec{r}_I} + E_{or} e^{j\omega t} e^{-j\vec{k}_r \cdot \vec{r}_I} = E_{ot} e^{j\omega t} e^{-j\vec{k}_t \cdot \vec{r}_I}$$

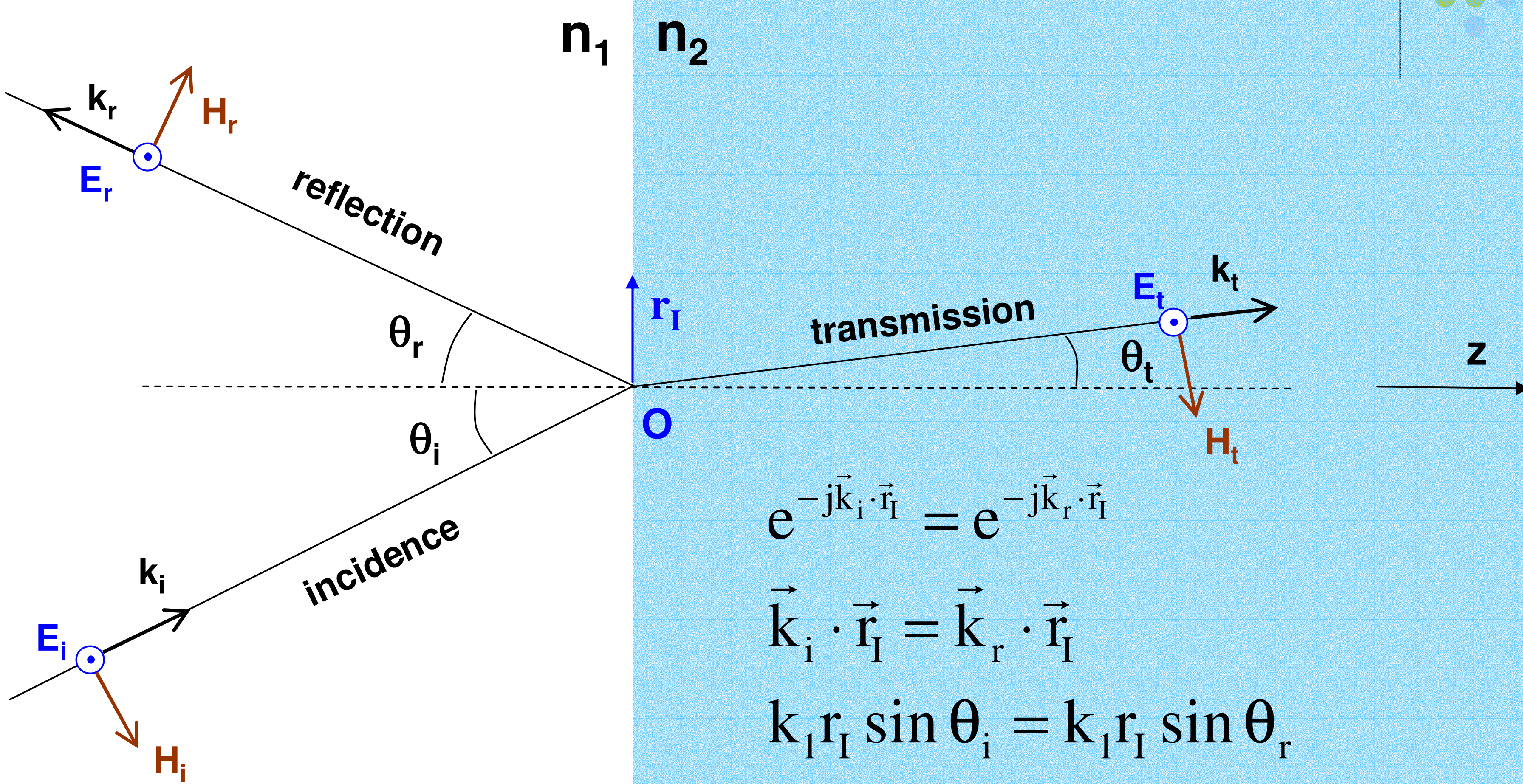
$$E_{oi} e^{-j\vec{k}_i \cdot \vec{r}_I} + E_{or} e^{-j\vec{k}_r \cdot \vec{r}_I} = E_{ot} e^{-j\vec{k}_t \cdot \vec{r}_I}$$

E_{oi}…etc are just numbers. True for ALL time r_I ?

$$e^{-j\vec{k}_i \cdot \vec{r}_I} = e^{-j\vec{k}_r \cdot \vec{r}_I} = e^{-j\vec{k}_t \cdot \vec{r}_I}$$

Reflection

⊥ polarization



$$e^{-j\vec{k}_i\cdot\vec{r}_I} = e^{-j\vec{k}_r\cdot\vec{r}_I}$$

$$\vec{k}_i \cdot \vec{r}_I = \vec{k}_r \cdot \vec{r}_I$$

$$k_1 r_I \sin\theta_i = k_1 r_I \sin\theta_r$$

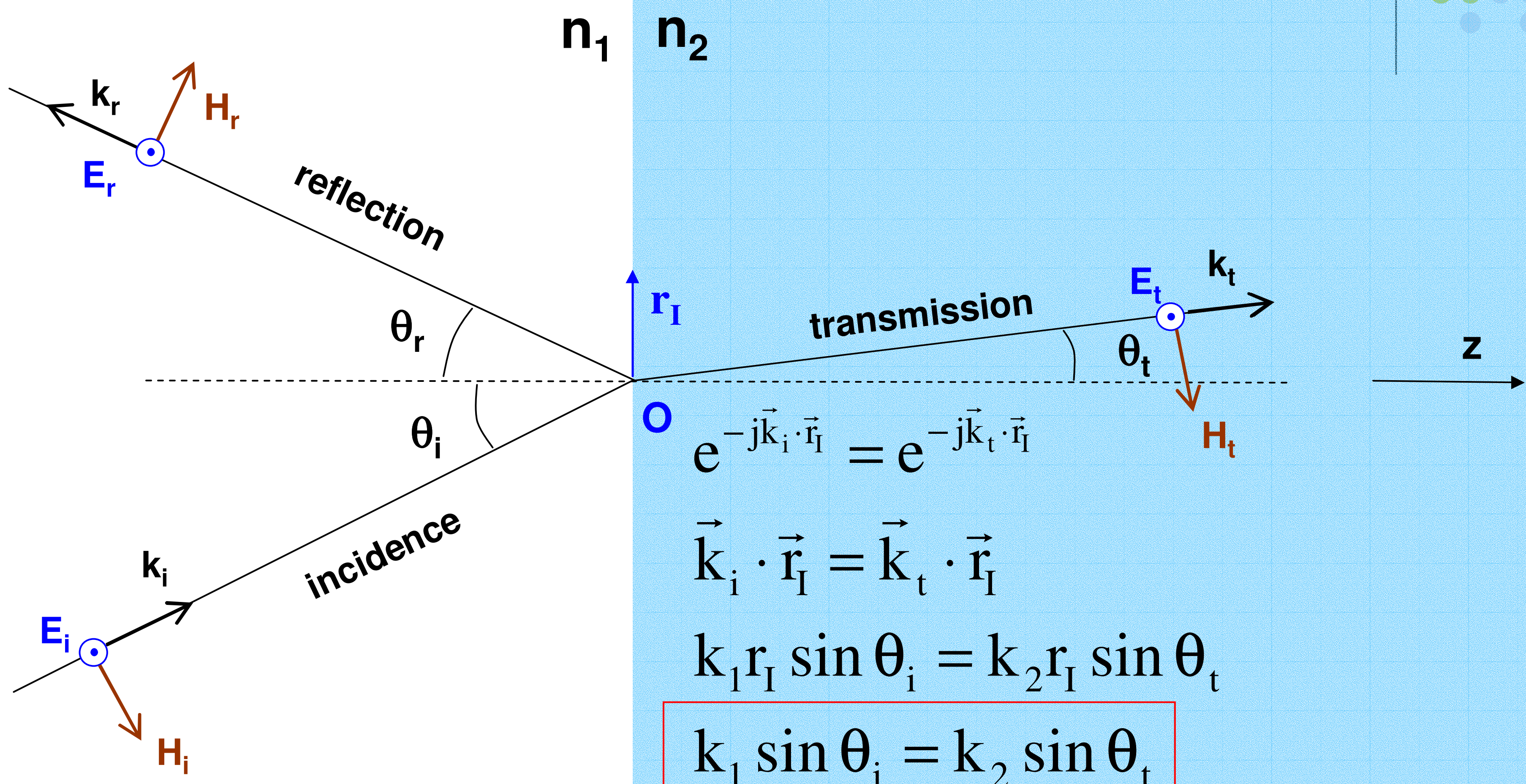
$$\sin\theta_i = \sin\theta_r$$

$$\boxed{\theta_i = \theta_r}$$

$$k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi n_1}{\lambda_o} = n_1 k_o = \frac{n_1\omega}{c}$$

Refraction

⊥ polarization



$$e^{-j\vec{k}_i \cdot \vec{r}_I} = e^{-j\vec{k}_t \cdot \vec{r}_I}$$

$$\vec{k}_i \cdot \vec{r}_I = \vec{k}_t \cdot \vec{r}_I$$

$$k_1 r_I \sin\theta_i = k_2 r_I \sin\theta_t$$

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

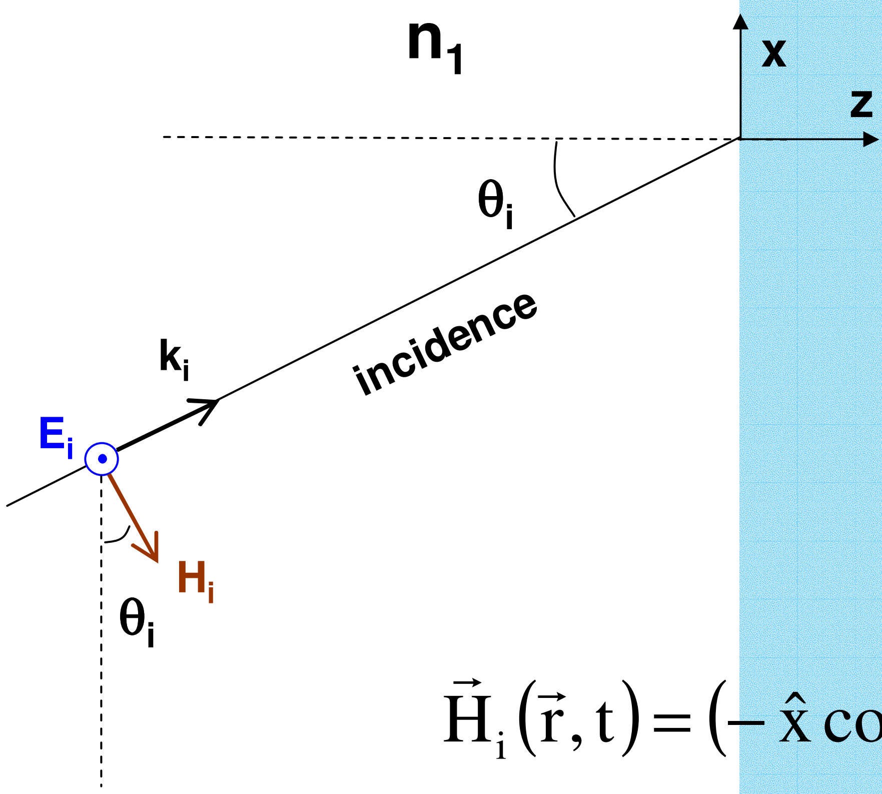
$$n_1 k_o \sin\theta_i = n_2 k_o \sin\theta_t$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

Snell's Law

Incidence waves

⊥ polarization



$$\vec{k}_i = k_1(\hat{x}\sin\theta_i + \hat{z}\cos\theta_i)$$

$$k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi n_1}{\lambda_o} = n_1 k_o = \frac{n_1\omega}{c}$$

$$\vec{E}_i(\vec{r},t) = \hat{y}E_{oi}e^{j(\omega t - \vec{k}_i\cdot\vec{r})}$$

$$\vec{E}_i(\vec{r},t) = \hat{y}E_{oi}e^{j(\omega t - k_1(x\sin\theta_i + z\cos\theta_i))}$$

$$\vec{H}_i(\vec{r},t) = (-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)\frac{E_{oi}}{\eta_1}e^{j(\omega t - k_1(x\sin\theta_i + z\cos\theta_i))}$$

Index of refraction $n_1 = \sqrt{\mu_{r1}\varepsilon_{r1}}$

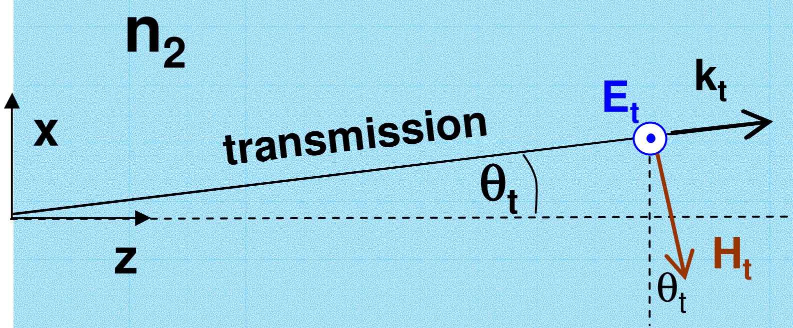
Wave impedance $\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_{r1}\mu_o}{\varepsilon_{r1}\varepsilon_o}} = \eta_o\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}$

Transmitted waves

⊥ polarization

$$\vec{k}_t = k_2\left(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t\right)$$

$$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi n_2}{\lambda_o} = n_2 k_o = \frac{n_2\omega}{c}$$



$$\vec{E}_t(\vec{r},t) = \hat{y}E_{ot}e^{j\left(\omega t - \vec{k}_t\cdot\vec{r}\right)}$$

$$\vec{E}_t(\vec{r},t) = \hat{y}E_{ot}e^{j\left(\omega t - k_2\left(x\sin\theta_t + z\cos\theta_t\right)\right)}$$

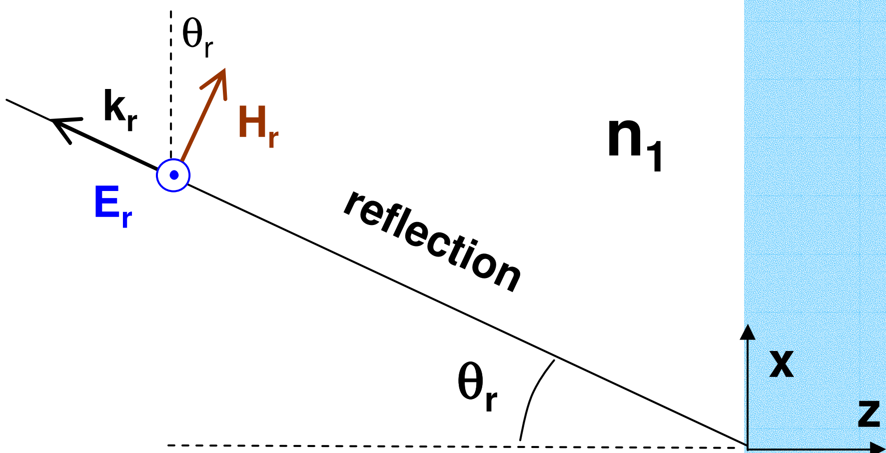
$$\vec{H}_t(\vec{r},t) = \left(-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t\right)\frac{E_{ot}}{\eta_2}e^{j\left(\omega t - k_2\left(x\sin\theta_t + z\cos\theta_t\right)\right)}$$

Index of refraction $n_2 = \sqrt{\mu_{r2}\varepsilon_{r2}}$

Wave impedance $\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_{r2}\mu_o}{\varepsilon_{r2}\varepsilon_o}} = \eta_o\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}$

Reflected waves

⊥ polarization



$$\vec{k}_r = k_1(\hat{x}\sin\theta_r - \hat{z}\cos\theta_r)$$

$$k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi n_1}{\lambda_o} = n_1 k_o = \frac{n_1\omega}{c}$$

$$\vec{E}_r(\vec{r},t) = \hat{y}E_{or}e^{j(\omega t - \vec{k}_r\cdot\vec{r})}$$

$$\vec{E}_r(\vec{r},t) = \hat{y}E_{or}e^{j(\omega t - k_1(x\sin\theta_r - z\cos\theta_r))}$$

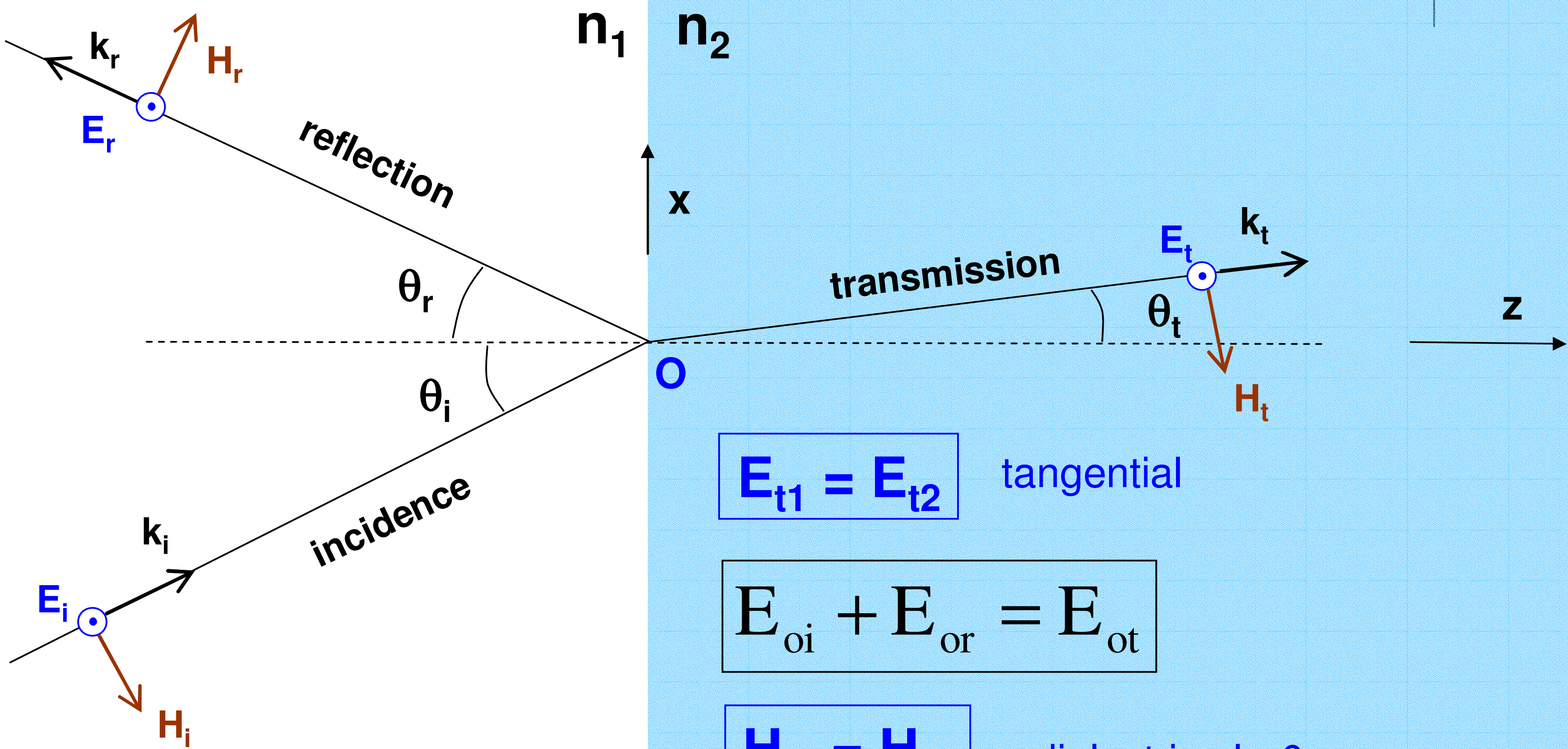
$$\vec{H}_r(\vec{r},t) = (\hat{x}\cos\theta_r + \hat{z}\sin\theta_r)\frac{E_{or}}{\eta_1}e^{j(\omega t - k_1(x\sin\theta_r - z\cos\theta_r))}$$

Index of refraction $n_1 = \sqrt{\mu_{r1}\varepsilon_{r1}}$

Wave impedance $\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_{r1}\mu_o}{\varepsilon_{r1}\varepsilon_o}} = \eta_o\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}$

At origin

⊥ polarization



$E_{t1} = E_{t2}$ tangential

$$E_{oi} + E_{or} = E_{ot}$$

$H_{t1} = H_{t2}$ dielectric, $J_s=0$

$$H_{oi}\cos\theta_i - H_{or}\cos\theta_r = H_{ot}\cos\theta_t$$

⊥ polarization

$$H_{oi}\cos\theta_i - H_{or}\cos\theta_r = H_{ot}\cos\theta_t$$

$$\frac{E_{oi}}{\eta_1}\cos\theta_i - \frac{E_{or}}{\eta_1}\cos\theta_i = \frac{E_{ot}}{\eta_2}\cos\theta_t$$

$$E_{oi} + E_{or} = E_{ot}$$

$$\frac{1}{\eta_1}\left(E_{oi}\cos\theta_i - E_{or}\cos\theta_i\right) = \frac{E_{oi}+E_{or}}{\eta_2}\cos\theta_t$$

$$E_{oi}\left(\frac{\cos\theta_i}{\eta_1} - \frac{\cos\theta_t}{\eta_2}\right) = E_{or}\left(\frac{\cos\theta_t}{\eta_2} + \frac{\cos\theta_i}{\eta_1}\right)$$

$$E_{oi}\left(\eta_2\cos\theta_i - \eta_1\cos\theta_t\right) = E_{or}\left(\eta_1\cos\theta_t + \eta_2\cos\theta_i\right)$$

$$\frac{E_{or}}{E_{oi}} \equiv \boxed{\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

Reflection coefficient

$$E_{ot} = E_{oi} + E_{or}$$

$$\frac{E_{ot}}{E_{oi}} = 1 + \frac{E_{or}}{E_{oi}}$$

$$\boxed{\tau_\perp = 1 + \Gamma_\perp}$$

Transmission coefficient

$$1 + \Gamma_\perp = 1 + \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\boxed{\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

Fresnel's Equations 1 & 2

$$n = \sqrt{\mu_r \varepsilon_r}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o \mu_r}{\sqrt{\varepsilon_r \mu_r}} = \frac{\eta_o \mu_r}{n}$$

n ~ mass density
= 1.00 air
= 1.33 water
= 1.5 glass
= 2.4 diamond

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = \frac{\frac{\eta_o \mu_{r2}}{n_2}\cos\theta_i - \frac{\eta_o \mu_{r1}}{n_1}\cos\theta_t}{\frac{\eta_o \mu_{r2}}{n_2}\cos\theta_i + \frac{\eta_o \mu_{r1}}{n_1}\cos\theta_t}$$

$$\Gamma_\perp = \frac{\frac{n_1}{\mu_{r1}}\cos\theta_i - \frac{n_2}{\mu_{r2}}\cos\theta_t}{\frac{n_1}{\mu_{r1}}\cos\theta_i + \frac{n_2}{\mu_{r2}}\cos\theta_t}$$

$$\tau_\perp = \frac{2\frac{n_1}{\mu_{r1}}\cos\theta_i}{\frac{n_1}{\mu_{r1}}\cos\theta_i + \frac{n_2}{\mu_{r2}}\cos\theta_t}$$

Non-magnetic materials

$$\Gamma_\perp = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

$$\tau_\perp = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

Non-magnetic materials $\perp$

$$\tau_\perp = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

τ is always > 0 , in-phase with incident

$$\Gamma_\perp = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

Γ can be in-phase or out-of-phase

Snell's $n_1 \sin\theta_i = n_2 \sin\theta_t$

IF $n_1 > n_2$

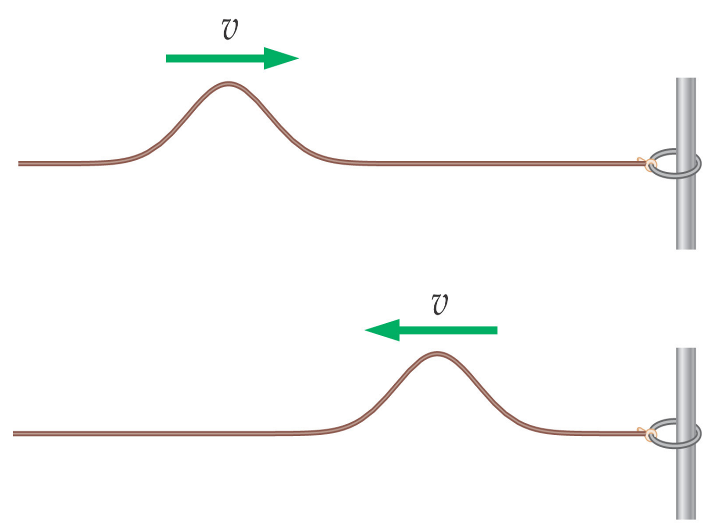
then $\theta_i < \theta_t$

$\cos\theta_i > \cos\theta_t$

$n_1 \cos\theta_i > n_2 \cos\theta_t$

$\Gamma_\perp > 0$

in-phase
e.g. glass-to-air



Non-magnetic ⊥ (continue)

$$\tau_\perp = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t}$$

$$\Gamma_\perp = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}$$

IF $n_1 < n_2$

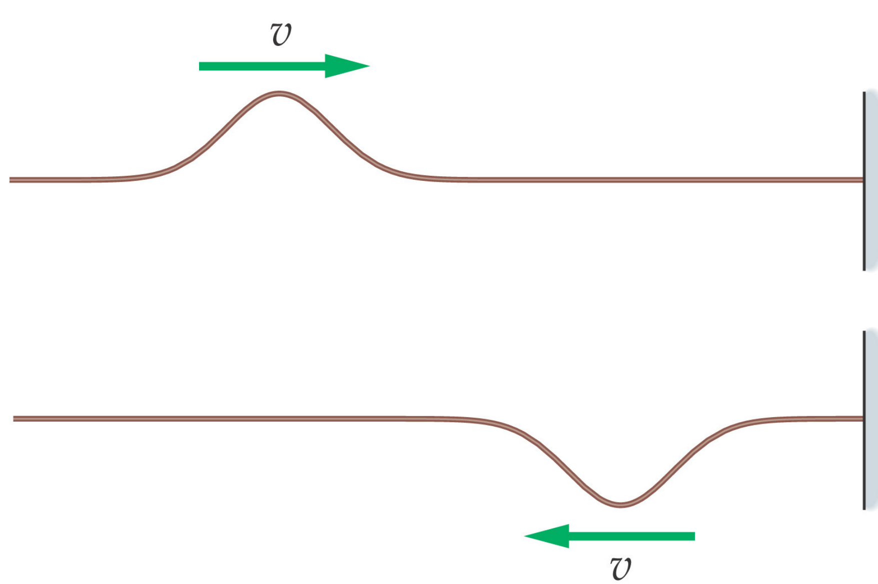
then $\theta_i > \theta_t$

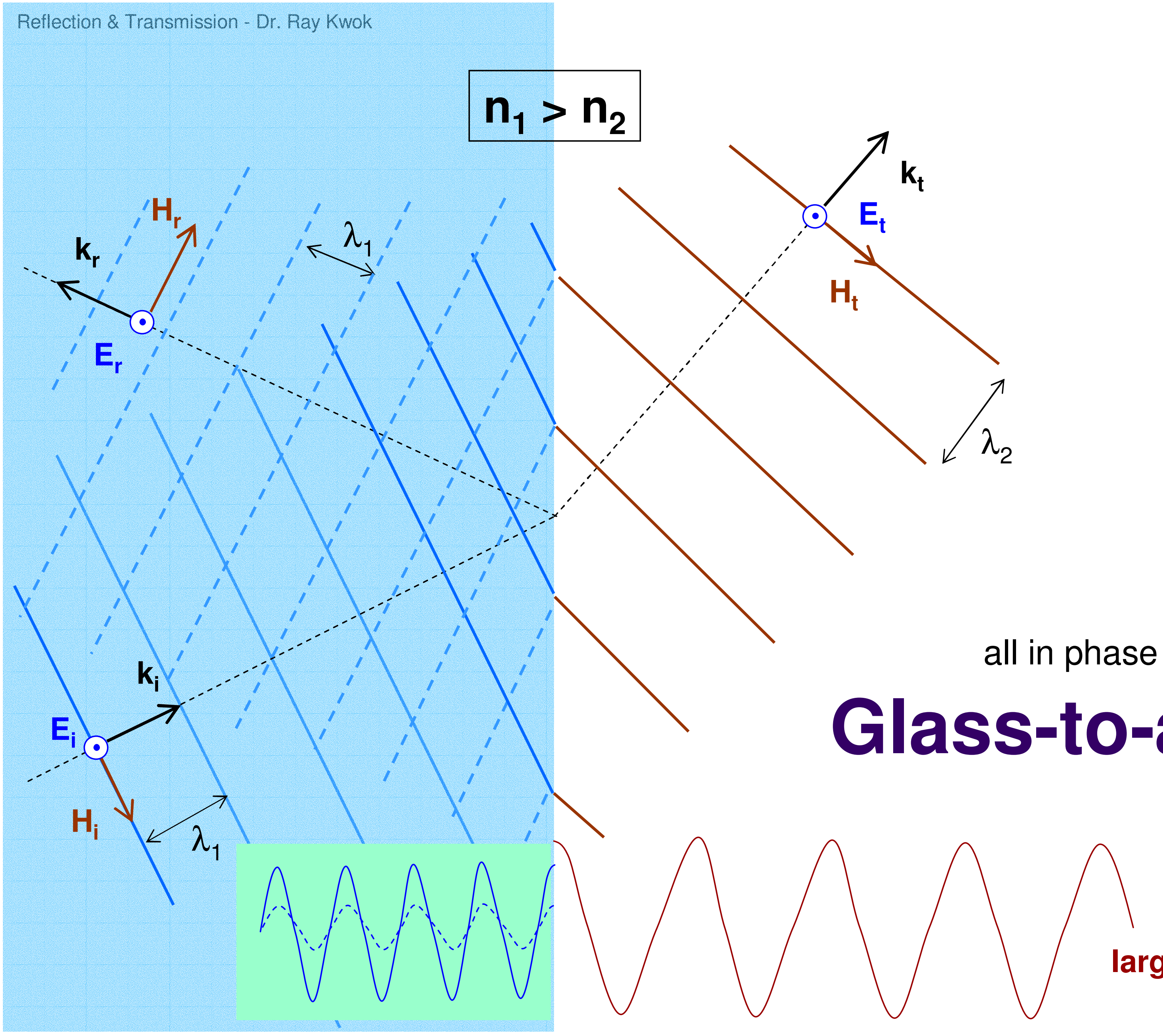
$\cos\theta_i < \cos\theta_t$

$n_1\cos\theta_i < n_2\cos\theta_t$

$\Gamma_\perp < 0$

180° out-of-phase
e.g. air-to-glass



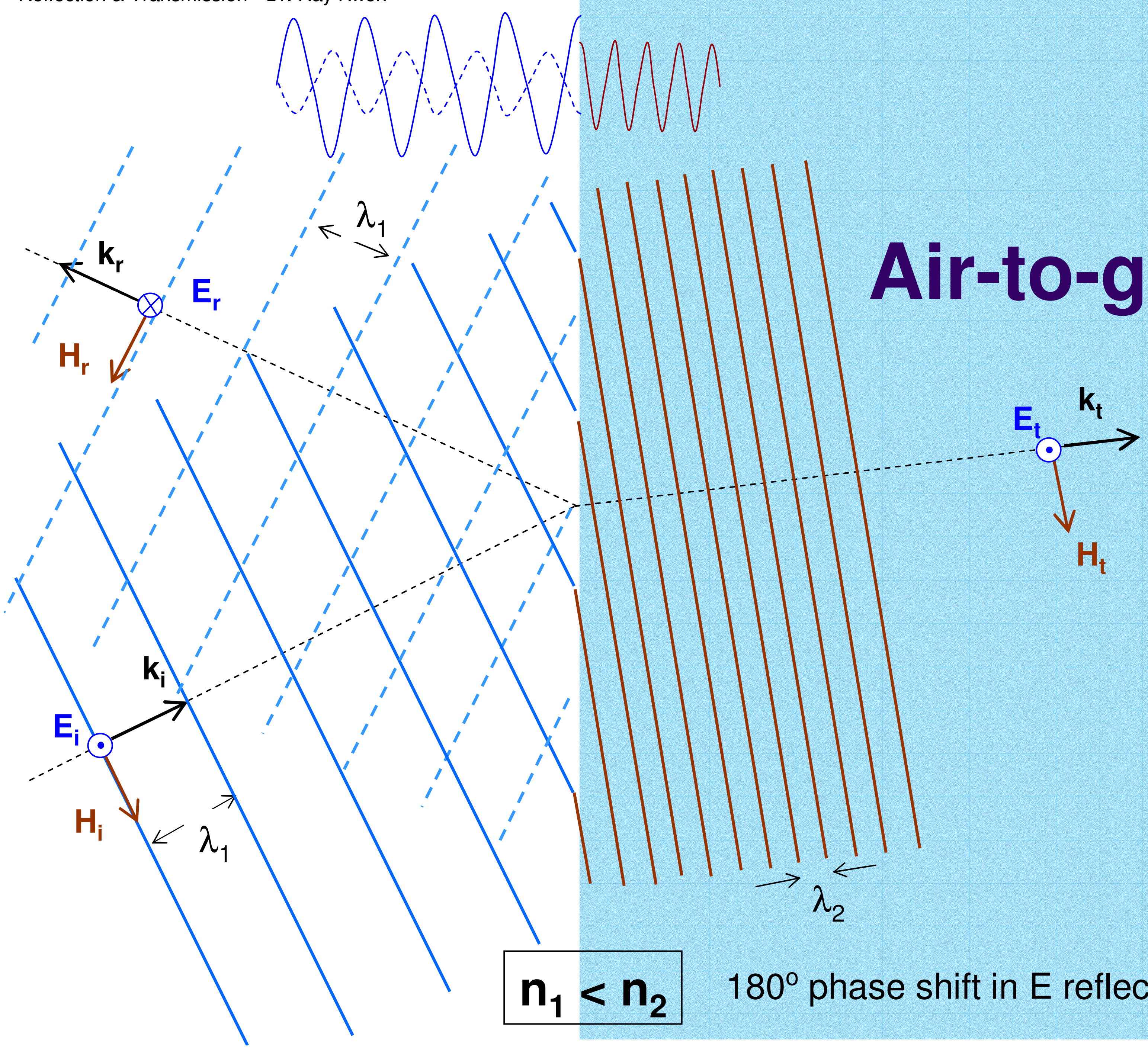


all in phase

Glass-to-air ⊥

larger amplitude?

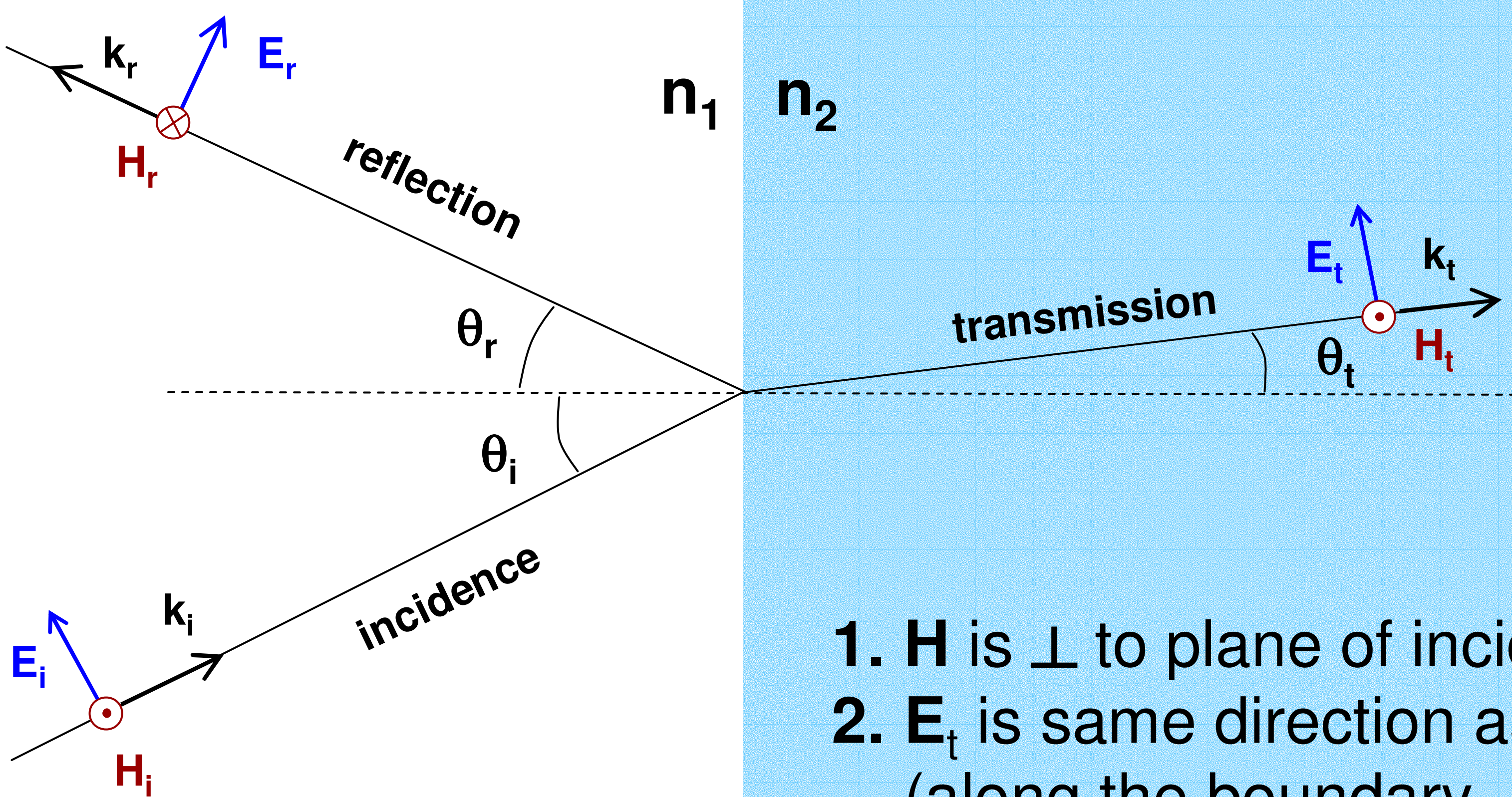
Air-to-glass ⊥



180° phase shift in E reflection

Parallel Polarization

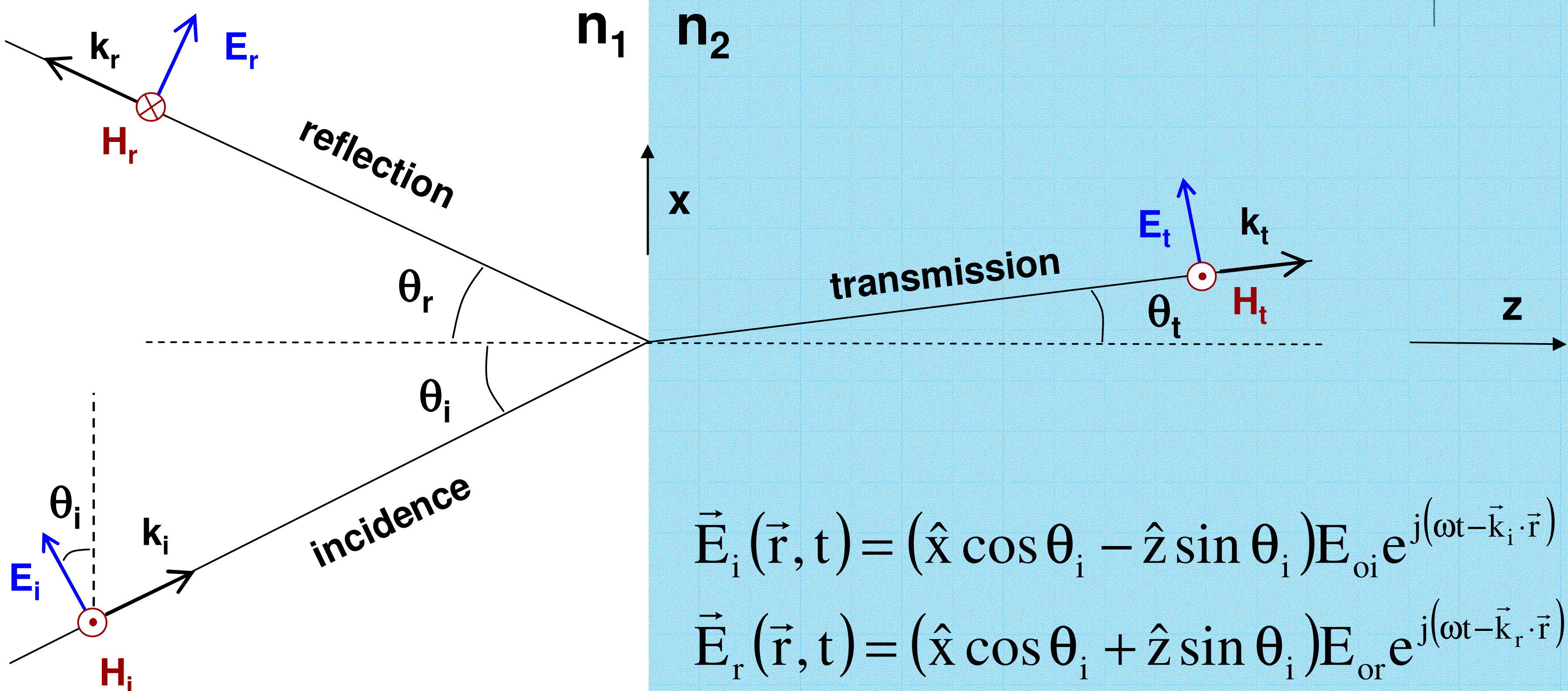
// to "plane of incidence"



1. **H** is ⊥ to plane of incidence
2. $\mathbf{E}_t$ is same direction as $\mathbf{E}_i$
   (along the boundary, later)
3. $\mathbf{E}_r$ can be up or down
4. **E** x **H** is along **k**

Wave Equations

// polarization



$$\vec{E}_i(\vec{r},t) = (\hat{x}\cos\theta_i - \hat{z}\sin\theta_i)E_{oi}e^{j(\omega t - \vec{k}_i\cdot\vec{r})}$$

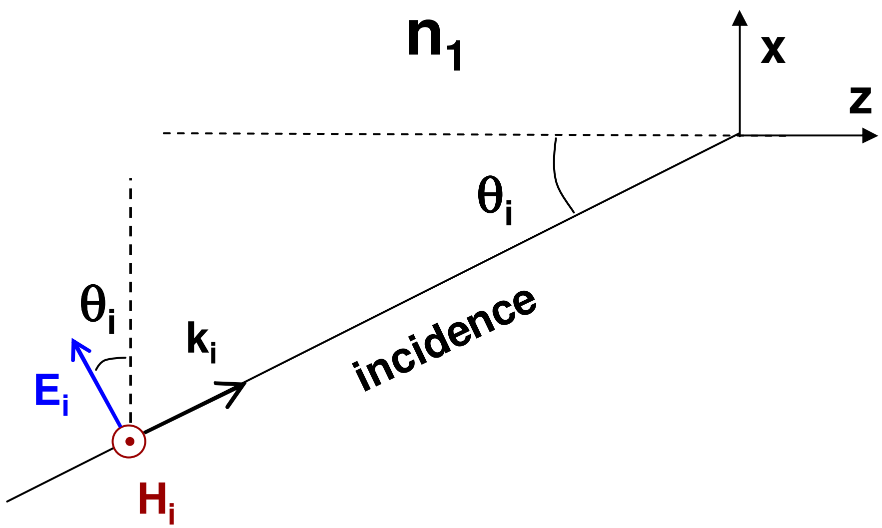
$$\vec{E}_r(\vec{r},t) = (\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)E_{or}e^{j(\omega t - \vec{k}_r\cdot\vec{r})}$$

$$\vec{E}_t(\vec{r},t) = (\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)E_{ot}e^{j(\omega t - \vec{k}_t\cdot\vec{r})}$$

What are the corresponding H-fields?

Incidence waves

// polarization



$$\vec{k}_i = k_1(\hat{x}\sin\theta_i + \hat{z}\cos\theta_i)$$

$$k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi n_1}{\lambda_o} = n_1 k_o = \frac{n_1\omega}{c}$$

$$\vec{E}_i(\vec{r},t) = (\hat{x}\cos\theta_i - \hat{z}\sin\theta_i)E_{oi}e^{j(\omega t - \vec{k}_i\cdot\vec{r})}$$

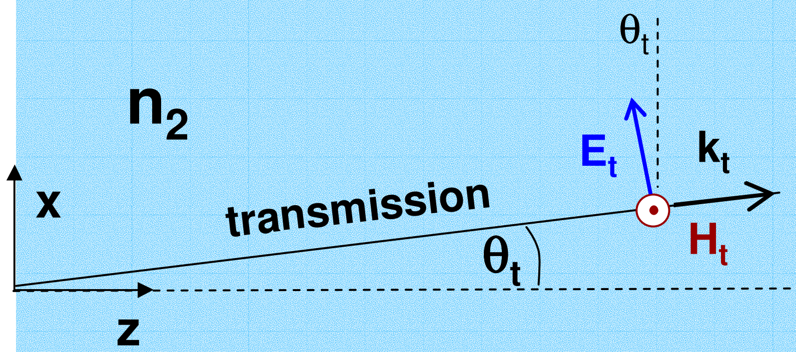
$$\vec{E}_i(\vec{r},t) = (\hat{x}\cos\theta_i - \hat{z}\sin\theta_i)E_{oi}e^{j(\omega t - k_1(x\sin\theta_i + z\cos\theta_i))}$$

$$\vec{H}_i(\vec{r},t) = \hat{y}\frac{E_{oi}}{\eta_1}e^{j(\omega t - k_1(x\sin\theta_i + z\cos\theta_i))}$$

Transmitted waves // polarization

$$\vec{k}_t = k_2(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t)$$

$$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi n_2}{\lambda_o} = n_2 k_o = \frac{n_2\omega}{c}$$



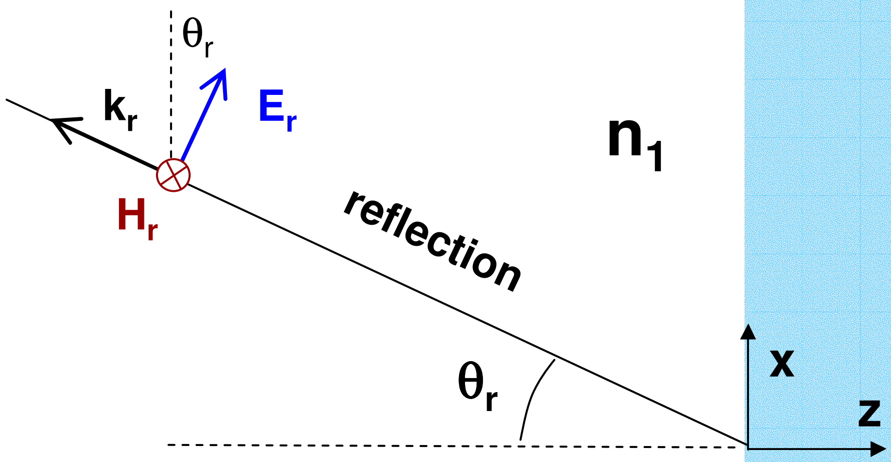
$$\vec{E}_t(\vec{r},t) = (\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)E_{ot}e^{j(\omega_t t - \vec{k}_t\cdot\vec{r})}$$

$$\vec{E}_t(\vec{r},t) = (\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)E_{ot}e^{j(\omega t - k_2(x\sin\theta_t + z\cos\theta_t))}$$

$$\vec{H}_i(\vec{r},t) = \hat{y}\frac{E_{ot}}{\eta_2}e^{j(\omega t - k_2(x\sin\theta_t + z\cos\theta_t))}$$

Reflected waves

// polarization



$$\vec{k}_r = k_1(\hat{x}\sin\theta_r - \hat{z}\cos\theta_r)$$

$$k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi n_1}{\lambda_o} = n_1 k_o = \frac{n_1\omega}{c}$$

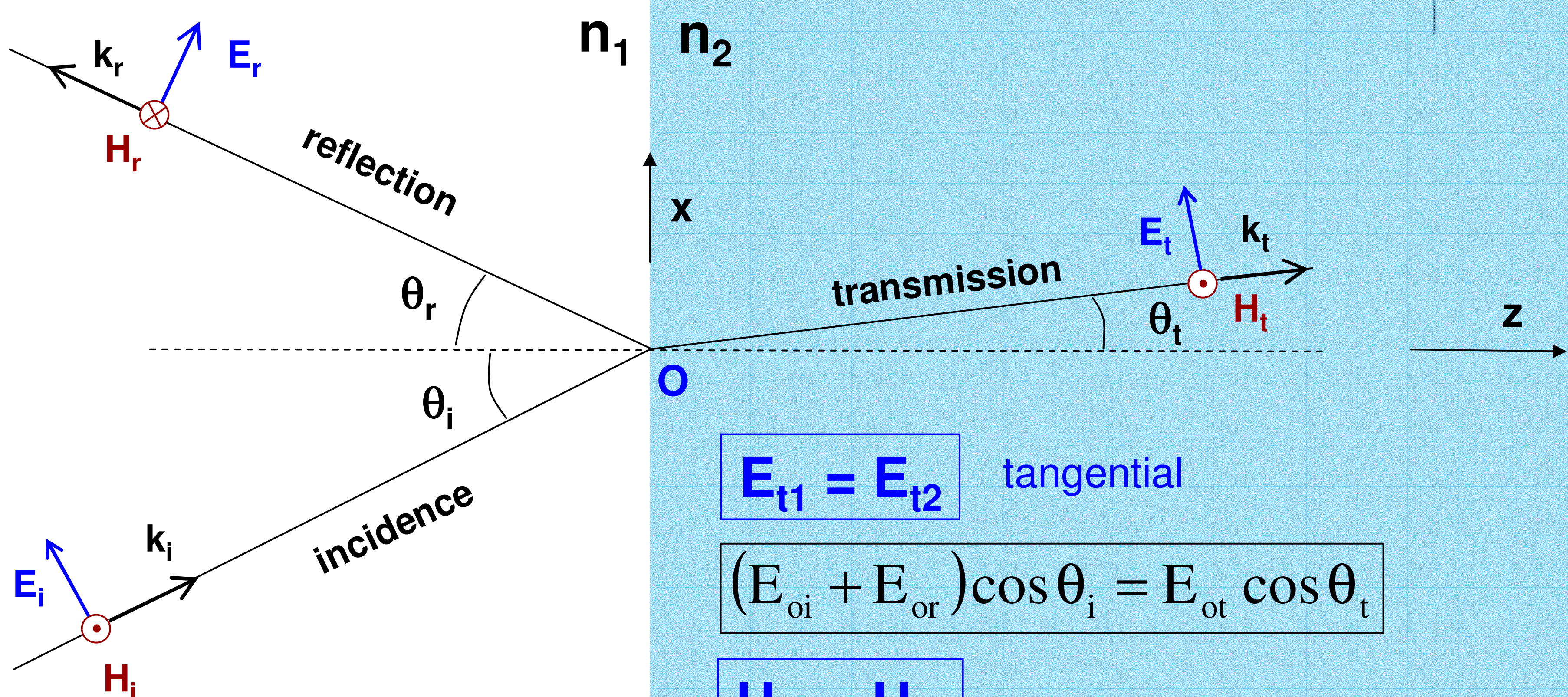
$$\vec{E}_r(\vec{r},t) = (\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)E_{or}e^{j(\omega_r t - \vec{k}_r\cdot\vec{r})}$$

$$\vec{E}_r(\vec{r},t) = (\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)E_{or}e^{j(\omega t - k_1(x\sin\theta_r - z\cos\theta_r))}$$

$$\vec{H}_r(\vec{r},t) = -\hat{y}\frac{E_{or}}{\eta_1}e^{j(\omega t - k_1(x\sin\theta_r - z\cos\theta_r))}$$

At origin

// polarization



$E_{t1} = E_{t2}$ tangential

$$\left(E_{oi} + E_{or}\right)\cos\theta_i = E_{ot}\cos\theta_t$$

$H_{t1} = H_{t2}$

$$\frac{E_{oi} - E_{or}}{\eta_1} = \frac{E_{ot}}{\eta_2}$$

// polarization

$$(E_{oi} + E_{or})\cos\theta_i = E_{ot}\cos\theta_t$$

and

$$\frac{E_{oi} - E_{or}}{\eta_1} = \frac{E_{ot}}{\eta_2}$$

$$(E_{oi} + E_{or})\cos\theta_i = \left(\frac{E_{oi} - E_{or}}{\eta_1}\right)\eta_2\cos\theta_t$$

$$E_{or}(\eta_1\cos\theta_i + \eta_2\cos\theta_t) = E_{oi}(\eta_2\cos\theta_t - \eta_1\cos\theta_i)$$

$$\frac{E_{or}}{E_{oi}} = \boxed{\frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \equiv \Gamma_{//}}$$

Reflection coefficient

$$\frac{E_{oi} - E_{or}}{\eta_1} = \frac{E_{ot}}{\eta_2}$$

$$\frac{E_{ot}}{E_{oi}} = \frac{\eta_2}{\eta_1}\left(1 - \frac{E_{or}}{E_{oi}}\right)$$

$$\frac{E_{ot}}{E_{oi}} = \frac{\eta_2}{\eta_1}\left(1 - \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}\right)$$

$$\frac{E_{ot}}{E_{oi}} = \frac{\eta_2}{\eta_1}\left(\frac{2\eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}\right)$$

$$\frac{E_{ot}}{E_{oi}} = \boxed{\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \equiv \tau_{//}}$$

Transmission coefficient

note

$$\tau_{//} = \frac{\eta_2}{\eta_1}(1 - \Gamma_{//})$$

$$\tau_{//} = \frac{\cos\theta_i}{\cos\theta_t}(1 + \Gamma_{//})$$

Fresnel's Equations 3 & 4

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o \mu_r}{\sqrt{\varepsilon_r \mu_r}} = \frac{\eta_o \mu_r}{n}$$

$$\Gamma_{//} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\Gamma_{//} = \frac{\frac{n_1}{\mu_{r1}}\cos\theta_t - \frac{n_2}{\mu_{r2}}\cos\theta_i}{\frac{n_1}{\mu_{r1}}\cos\theta_t + \frac{n_2}{\mu_{r2}}\cos\theta_i}$$

$$\tau_{//} = \frac{2\frac{n_1}{\mu_{r1}}\cos\theta_i}{\frac{n_1}{\mu_{r1}}\cos\theta_t + \frac{n_2}{\mu_{r2}}\cos\theta_i}$$

Non-magnetic materials

$$\Gamma_{//} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

$$\tau_{//} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

Non-magnetic materials //

$$\tau_{//} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

τ is always > 0 , in-phase with incident

$$\Gamma_{//} = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

Γ can be in-phase or out-of-phase

for $\Gamma_{//} > 0$

$$n_1 \cos\theta_t > n_2 \cos\theta_i$$

$$\frac{n_1}{n_2}\cos\theta_t > \cos\theta_i$$

$$\frac{\sin\theta_t}{\sin\theta_i}\cos\theta_t > \cos\theta_i$$

$$\sin\theta_t \cos\theta_t > \sin\theta_i \cos\theta_i$$

$$\sin 2\theta_t - \sin 2\theta_i > 0$$

$$\sin(\theta_t - \theta_i)\cos(\theta_t + \theta_i) > 0$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

2 solutions: $\theta_t > \theta_i$ & $\theta_t + \theta_i < \pi/2$ $(n_1 > n_2)$

$\theta_t < \theta_i$ & $\theta_t + \theta_i > \pi/2$ $(n_1 < n_2)$

in-phase

Brewster Angle

no reflection $\Gamma_{//} = 0$

when $\theta_i + \theta_t = \dfrac{\pi}{2}$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

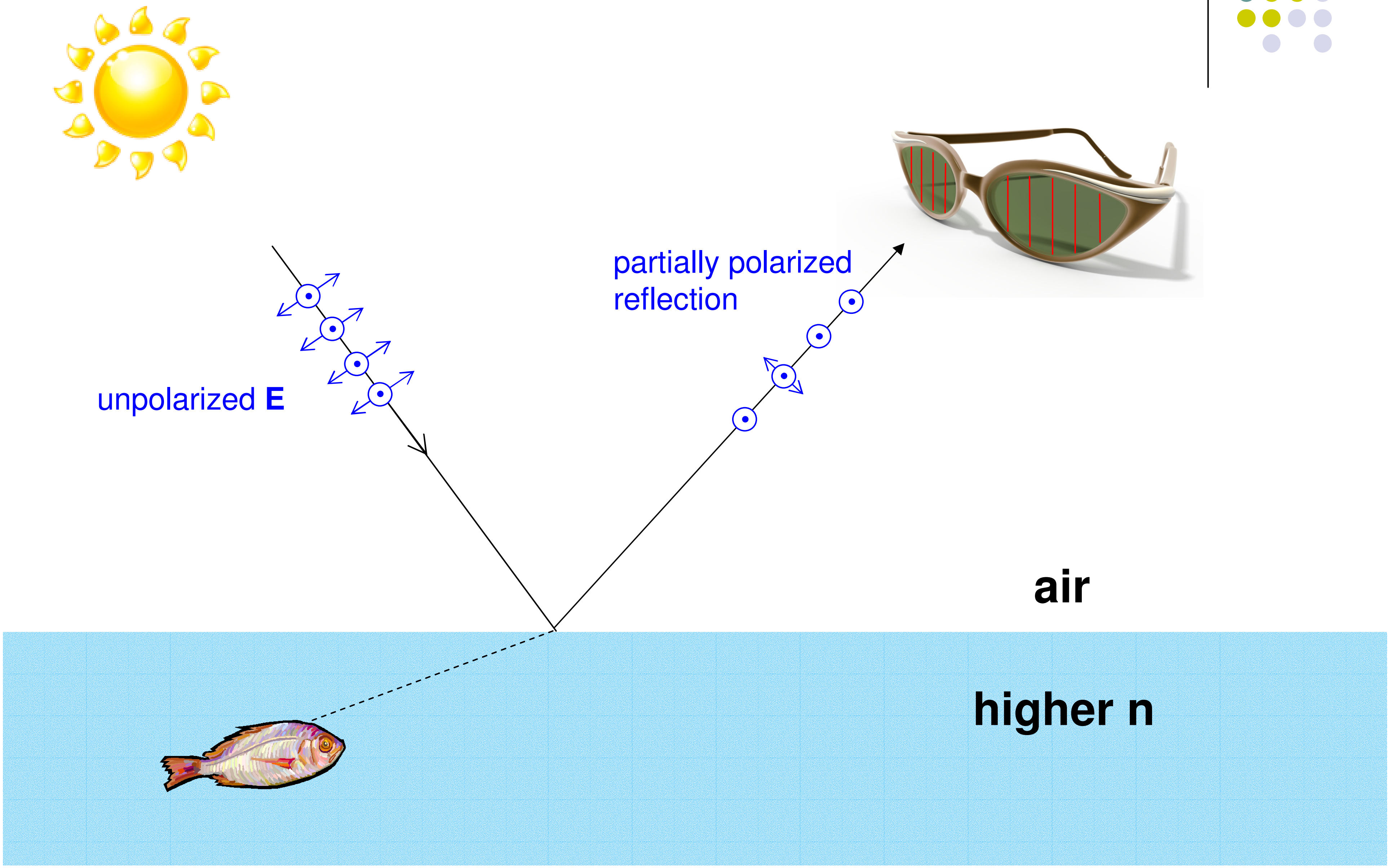
$$n_1 \sin\theta_i = n_2 \sin\left(\frac{\pi}{2} - \theta_i\right)$$

$$n_1 \sin\theta_i = n_2 \cos\theta_i$$

$$\tan\theta_i = \boxed{\frac{n_2}{n_1} \equiv \tan\theta_B}$$

A good way to measure dielectric constant remotely & non-destructively !!

Polarized Sunglasses



Brewster vs. Critical angle

Brewster Angle

Total transmission (no reflection)

Regardless $n_1 > n_2$ or $n_2 > n_1$

Only for //-polarization

$$\tan\theta_B = \frac{n_2}{n_1}$$

$$\theta_B + \theta_t = \frac{\pi}{2}$$

Critical Angle

Total reflection (no transmission)

Only for $n_1 > n_2$

Regardless $\perp$ or // – polarization

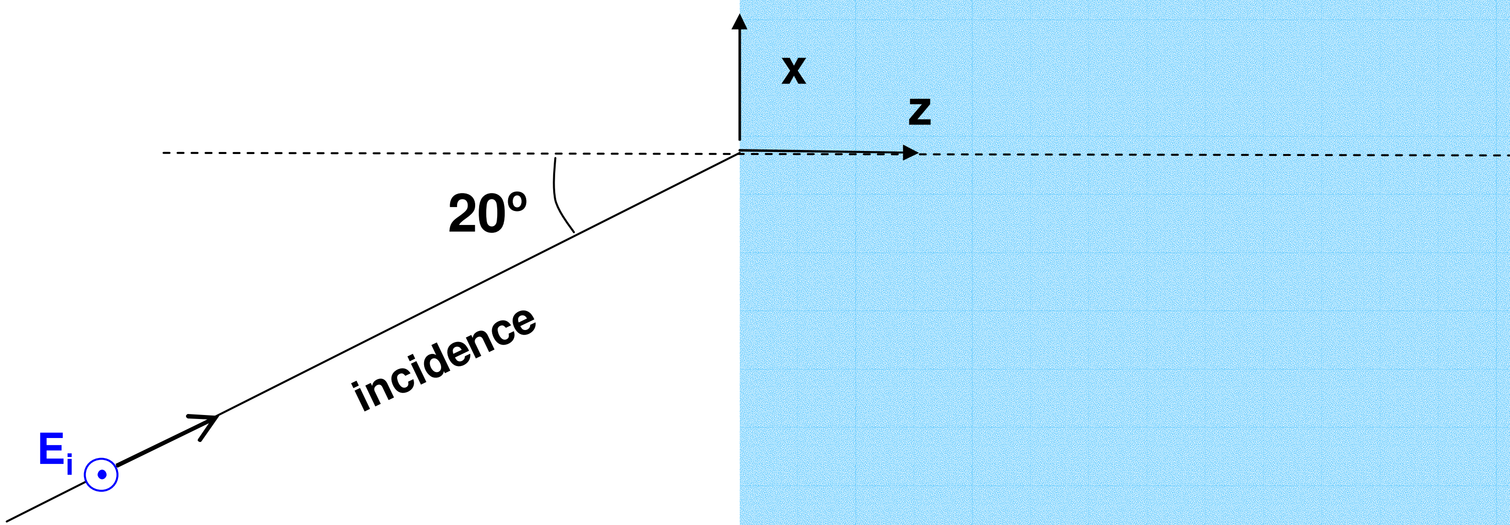
$$\sin\theta_C = \frac{n_2}{n_1}$$

Exercise

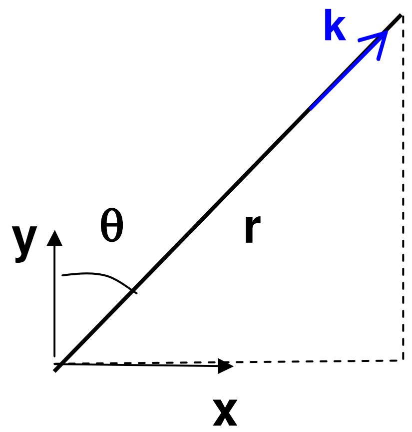
E_{oi} = 2 V/m
f = 159 MHz
Find E_r and H_t ?

air

glass
n = 1.5



Attenuation term



How to express $e^{-\alpha r}$ term??

$$\hat{k} = \hat{x}\sin\theta + \hat{y}\cos\theta$$

$$e^{-\alpha r} = e^{-\alpha(x\sin\theta + y\cos\theta)} = e^{-\alpha\sqrt{x^2+y^2}}$$

Same ?

$$x\sin\theta + y\cos\theta = x\left(\frac{x}{r}\right) + y\left(\frac{y}{r}\right) = \frac{x^2+y^2}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2}$$

Yes, Q.E.D.

Can think of :

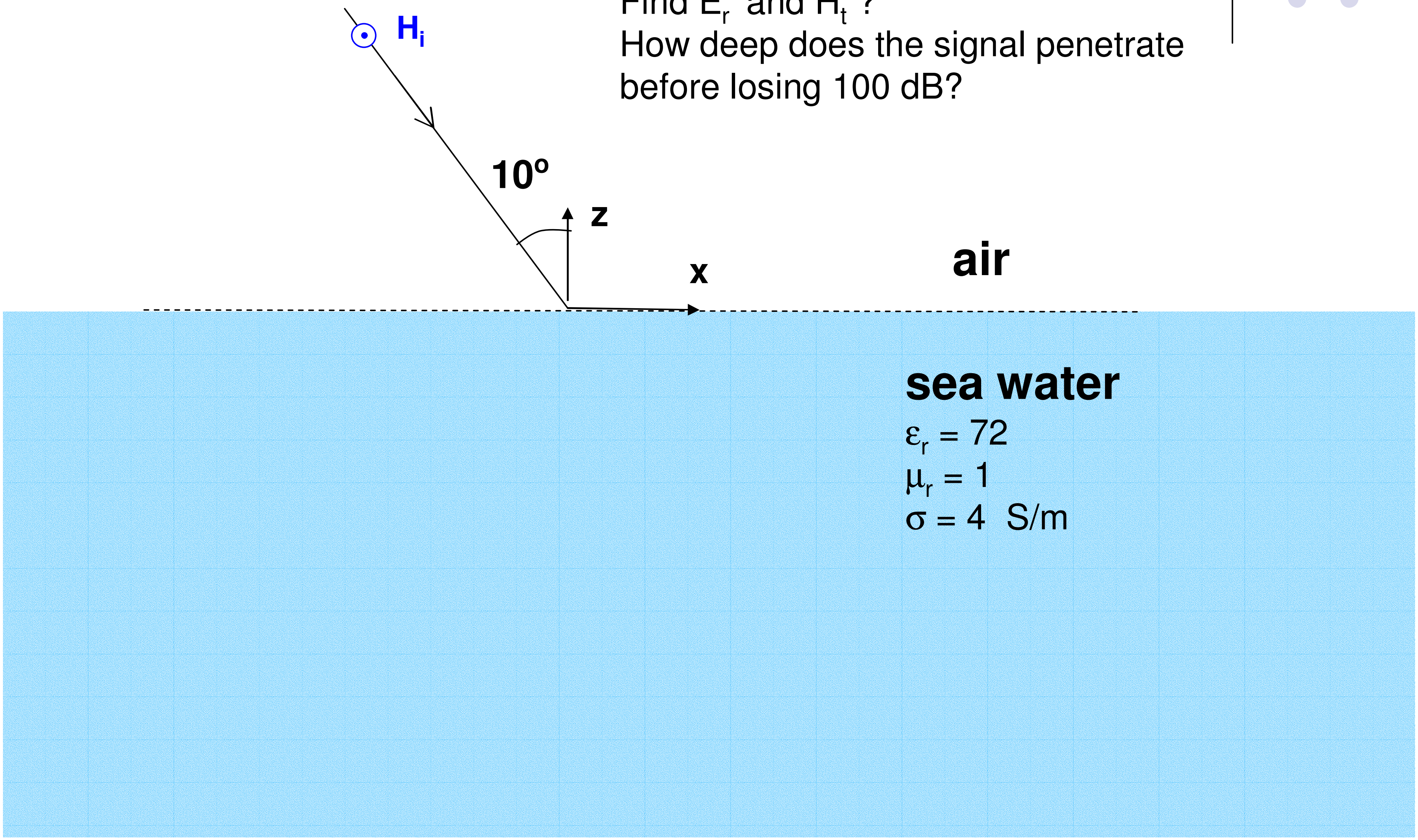
$$\vec{\alpha} \equiv \alpha\hat{k}$$

$$\vec{\alpha}\cdot\vec{r} = \alpha(x\sin\theta + y\cos\theta)$$

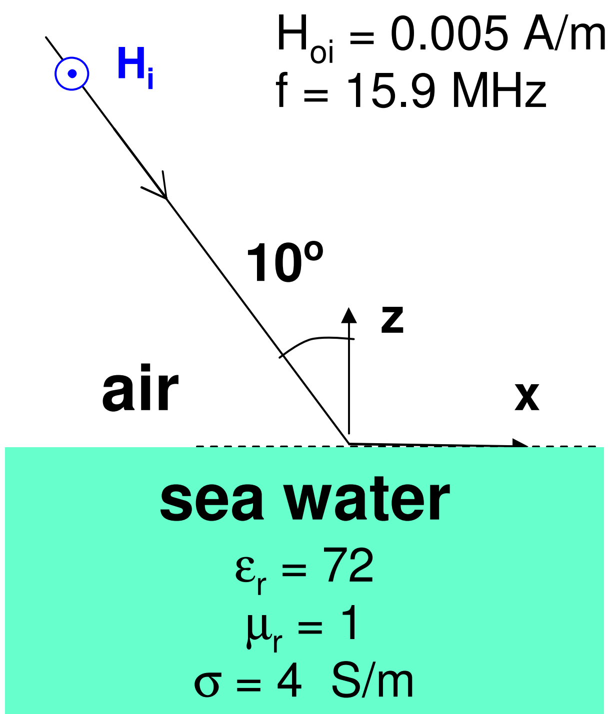
$$e^{-\vec{\alpha}\cdot\vec{r}} = e^{-\alpha(x\sin\theta + y\cos\theta)} = e^{-\alpha\sqrt{x^2+y^2}}$$

Exercise

H_{oi} = 0.005 A/m
f = 15.9 MHz
Find E_r and H_t ?
How deep does the signal penetrate before losing 100 dB?



Answer



// - polarization

$\omega = 2\pi f = 10^8$ rad/s

$$\tan\delta \equiv \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{4}{\left(10^8\right)\left(\frac{72}{36\pi}\cdot 10^{-9}\right)} = 63$$

$$\alpha^2 = \frac{\omega^2\mu\varepsilon'}{2}\left(\sqrt{1+\tan^2\delta}-1\right)$$

$$\alpha^2 = \frac{\left(10^8\right)^2\left(4\pi\cdot 10^{-7}\right)\left(\frac{72}{36\pi}\cdot 10^{-9}\right)}{2}\left(\sqrt{1+63^2}-1\right) = 4(63-1)$$

$$\alpha = 15.8$$

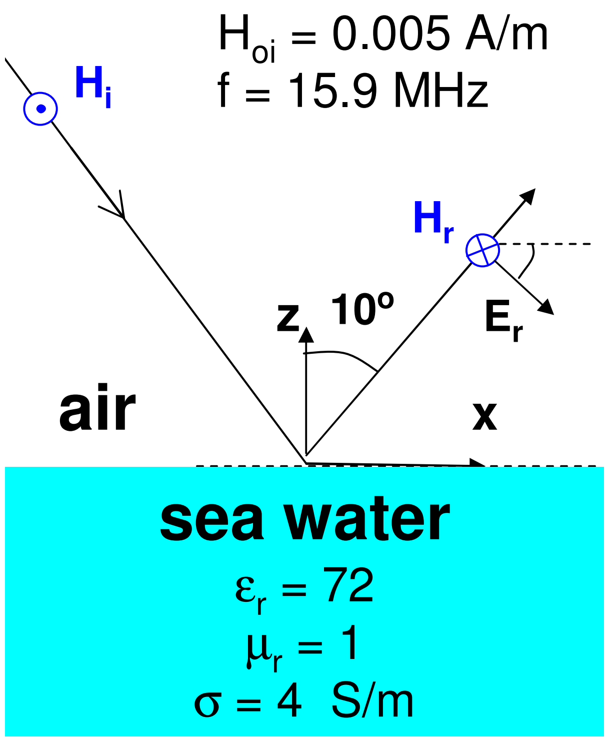
$$\beta^2 = \frac{\omega^2\mu\varepsilon'}{2}\left(\sqrt{1+\tan^2\delta}+1\right) = 4(63+1)$$

$$\beta = 16$$

$\alpha \approx \beta = 16$ ~ good conductor

$$\eta_2 = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{16}{4} = 4(1+j) = 5.7\angle 45^\circ$$

Continue..



$$E_{oi} = \eta_o H_{oi} = 377(0.005) = 1.89$$

$$k_1 = \frac{\omega}{c} = \frac{10^8}{3 \cdot 10^8} = \frac{1}{3} = 0.33$$

$$\vec{k}_r = k_1(\hat{z}\cos\theta_i + \hat{x}\sin\theta_i) = 0.328\hat{z} + 0.058\hat{x}$$

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$ Snell's Law

$$(0.33)\sin(10^o) = (16)\sin\theta_t$$

$$\theta_t = 0.2^o$$

$$\Gamma_{//} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

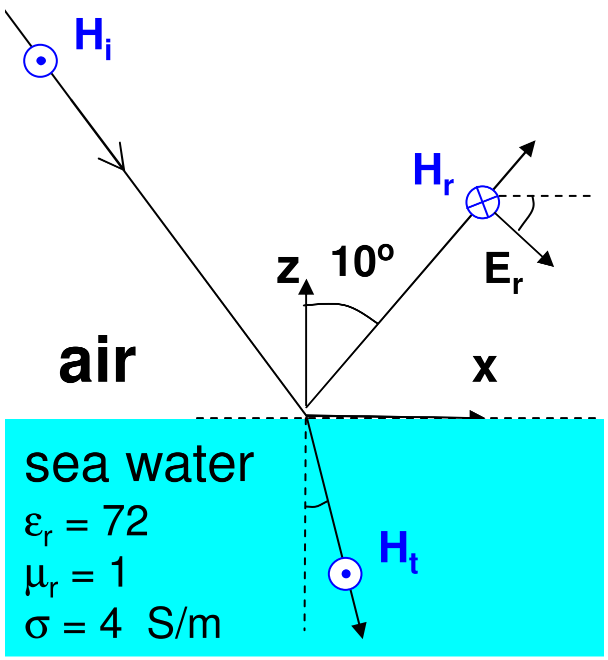
$$\Gamma_{//} = \frac{4(1+j)\cos(0.2^o) - 377\cos(10^o)}{4(1+j)\cos(0.2^o) - 377\cos(10^o)} = 0.979\angle 179^o$$

$$\vec{E}_r = (\hat{x}\cos\theta_i - \hat{z}\sin\theta_i)(E_{oi}\Gamma_{//})\cos(\omega t - 0.328z - 0.058x)$$

$$\vec{E}_r = (\hat{x}\cos(10^o) - \hat{z}\sin(10^o))(1.89(0.979\angle 179^o))\cos(\omega t - 0.328z - 0.058x)$$

$$\vec{E}_r = (\hat{x}1.82 - \hat{z}0.32)\cos(10^8 t - 0.328z - 0.058x + 3.12)$$ V/m

Continue..



$$\tau_{//} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\tau_{//} = \frac{2(5.7\angle 45^o)\cos(10^o)}{4(1+j)\cos(0.2^o) - 377\cos(10^o)} = 0.030\angle 44^o$$

$$\vec{k}_t = \beta(\hat{x}\sin\theta_t - \hat{z}\cos\theta_t) = 16(\hat{x}\sin(0.2^o) - \hat{z}\cos(0.2^o))$$

$$\vec{k}_t = 0.06\hat{x} - 16\hat{z}$$

$$\vec{k}_t \cdot \vec{r} = 0.06x - 16z = \beta r = \alpha r$$

$$\vec{H}_t = (-\hat{y})\left(\frac{E_{oi}\tau_{//}}{\eta_2}\right)e^{-\alpha r}\cos(\omega t - \beta r)$$

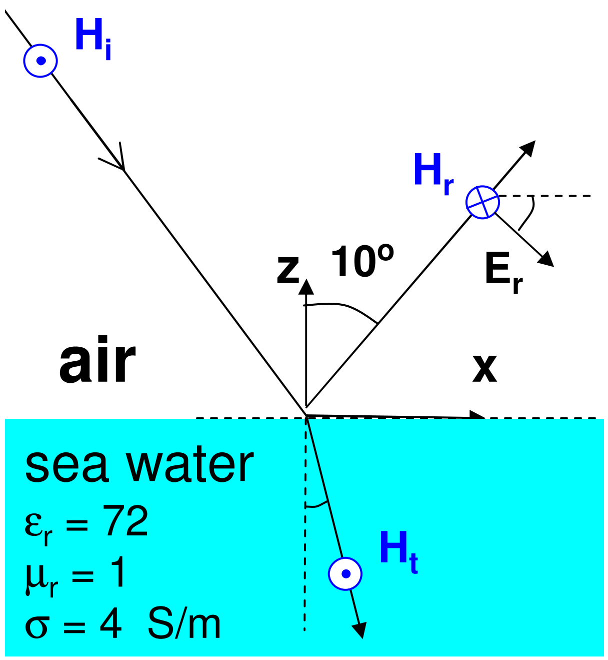
$$\frac{E_{oi}\tau_{//}}{\eta_2} = \frac{(1.89)(0.030\angle 44^o)}{5.7\angle 45^o} = 0.0099\angle -0.6^o$$

$$\vec{H}_t = -\hat{y}(0.0099\angle -0.6^o)e^{-0.06x+16z}\cos(\omega t - 0.06x + 16z)$$

$$\vec{H}_t = \hat{y}9.9e^{-0.06x+16z}\cos(10^8 t - 0.06x + 16z + 3.13) \quad \text{mA/m}$$

Continue..

How deep does the signal penetrate before losing 100 dB?



$$-100\text{dB} = -8.686\alpha r$$

$$\alpha = 16$$

$$r = 0.719$$

$$z = r\cos\theta_t = 0.719\cos(0.2^o) \approx 0.719 \approx 72\text{cm}$$

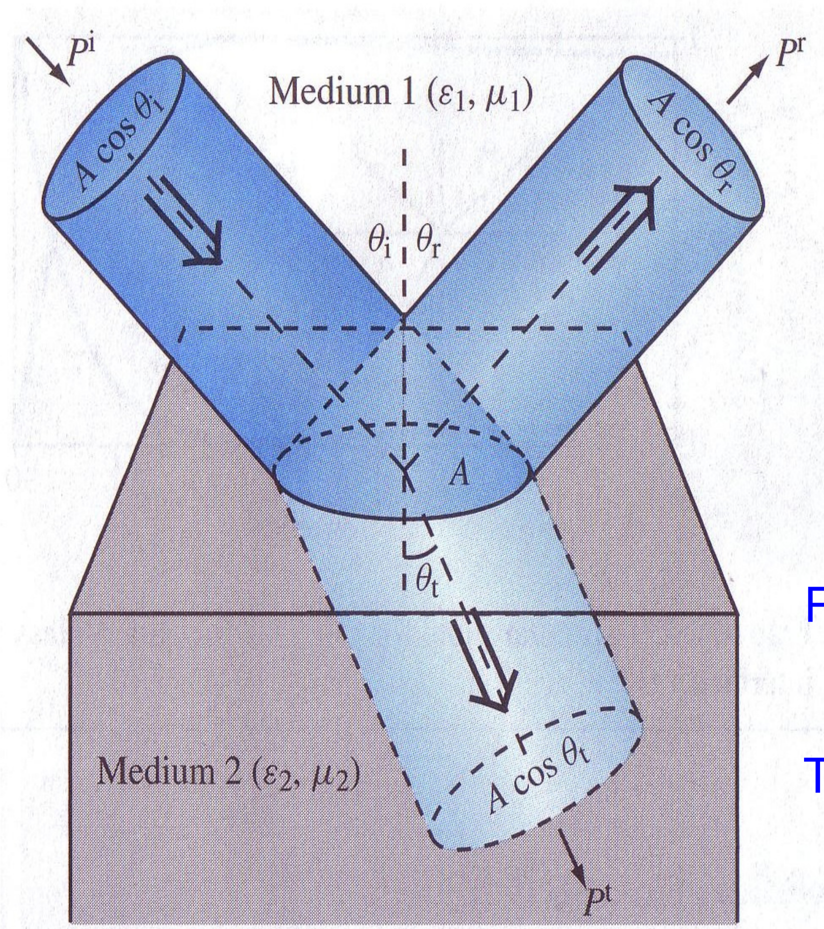
attenuate rapidly!!!

$$\delta_s = \frac{1}{\alpha} = \frac{1}{16} = 6.25\text{cm}$$

* it's difficult to do air-submarine communication !!

(optics)

Reflectance & Transmittance



Radiation Power

$$P_i = \int \vec{S}_i \cdot d\vec{a} = S_i A\cos\theta_i = \Re e\left\{\frac{|E_{oi}|^2}{2\eta_1} A\cos\theta_i\right\}$$

$$P_r = \int \vec{S}_r \cdot d\vec{a} = S_r A\cos\theta_r = \Re e\left\{\frac{|E_{or}|^2}{2\eta_1} A\cos\theta_i\right\}$$

$$P_t = \int \vec{S}_t \cdot d\vec{a} = S_t A\cos\theta_t = \Re e\left\{\frac{|E_{ot}|^2}{2\eta_2} A\cos\theta_t\right\}$$

Reflectance

$$R \equiv \frac{P_r}{P_i} = \frac{|E_{or}|^2 \cos\theta_i}{|E_{oi}|^2 \cos\theta_i} = |\Gamma|^2$$

Transmittance

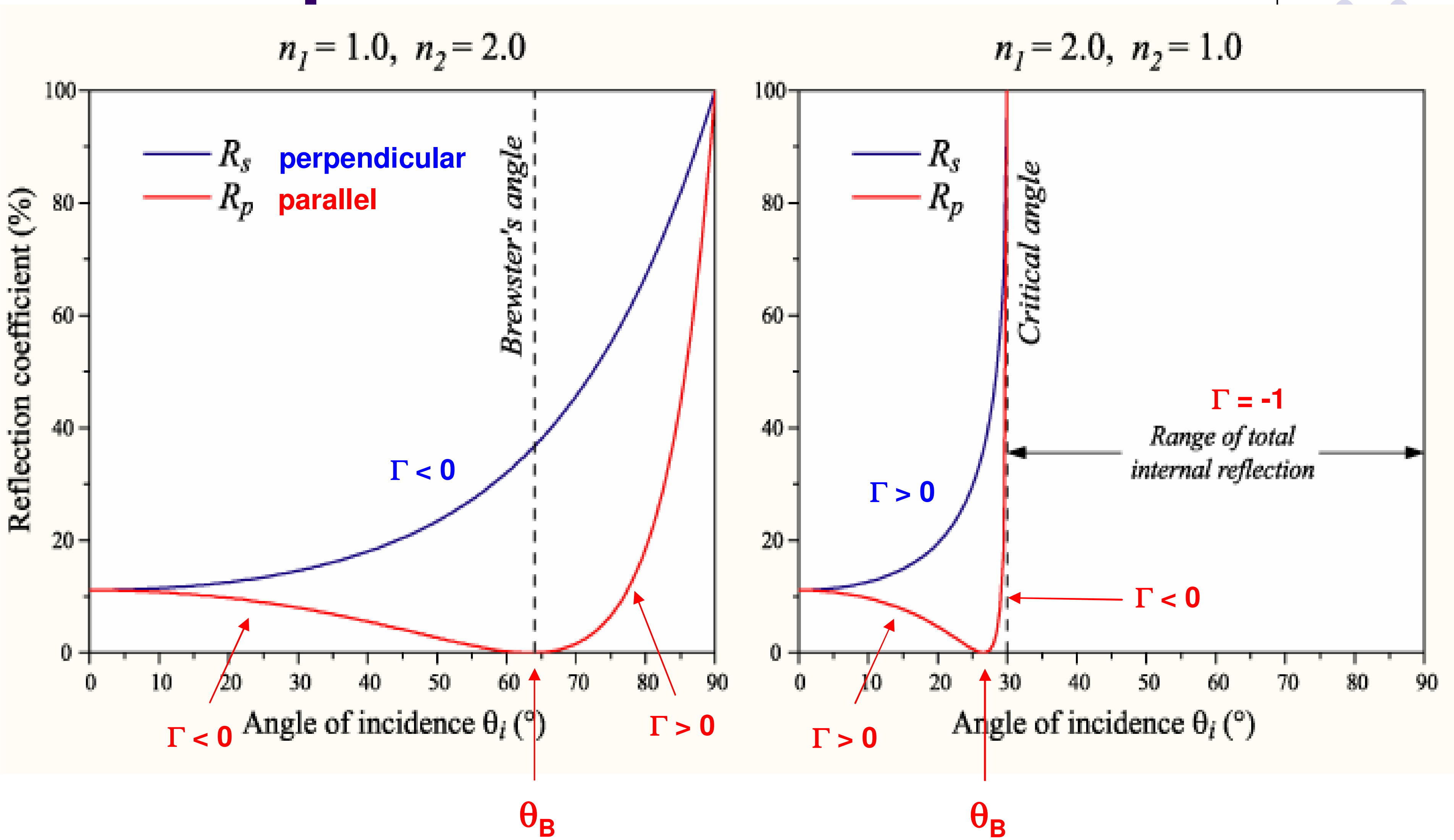
$$T = \frac{P_t}{P_i} = \frac{|E_{ot}|^2 \eta_1 \cos\theta_t}{|E_{oi}|^2 \eta_2 \cos\theta_i} = |\tau|^2 \left(\frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}\right)$$

$$P_i = P_r + P_t$$

$$\boxed{1 = R + T}$$

energy conservation

Example - Reflectance



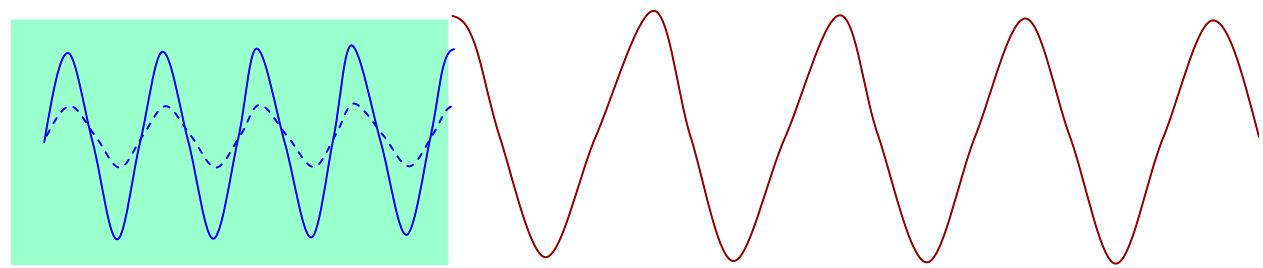
Normal incidence

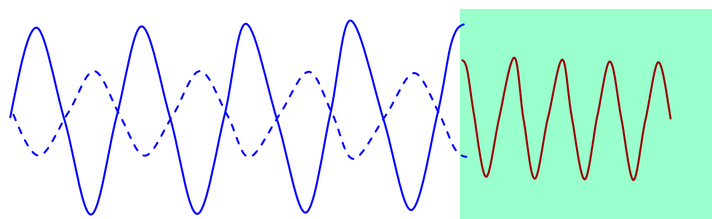
$$\theta_i = \theta_r = \theta_t = 0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

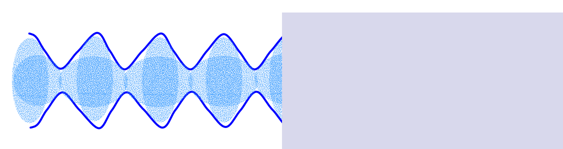
$n_1 > n_2$





$n_1 < n_2$

Standing wave



Standing Wave Ratio

$$SWR \equiv \frac{|E_{max}|}{|E_{min}|} = \frac{|E_{oi}| + |E_{or}|}{|E_{oi}| - |E_{or}|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

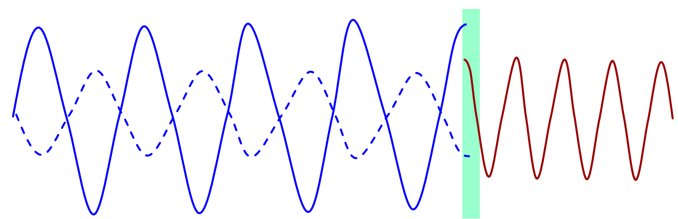
SWR = 1 (no reflection)

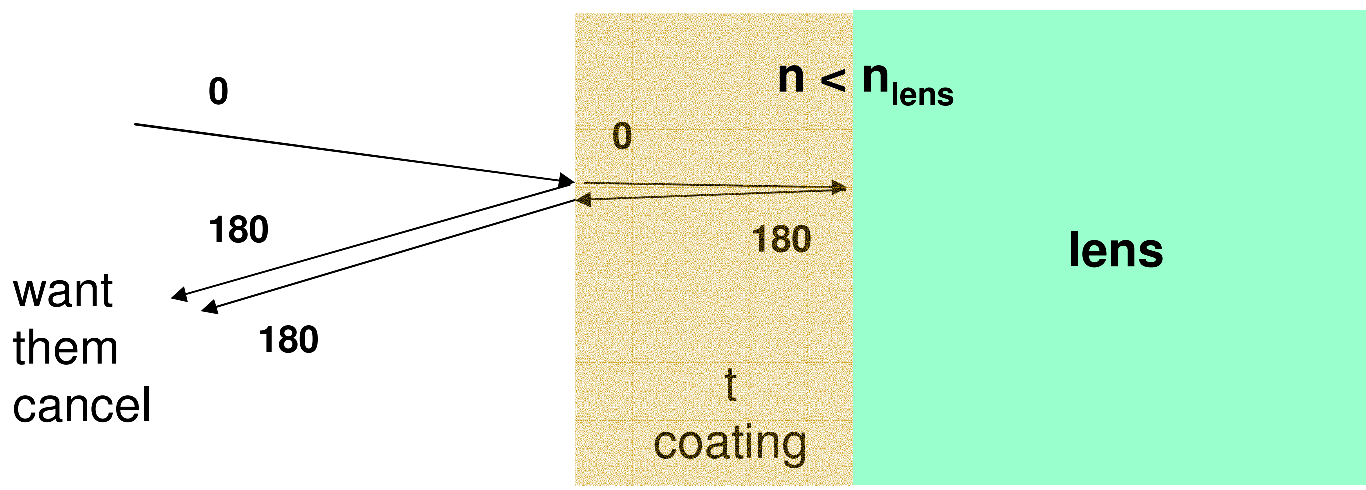
SWR→∞ (total reflection)

Anti-reflection coating

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{n_1 - n_2}{n_1 + n_2}$$

non-magnetic



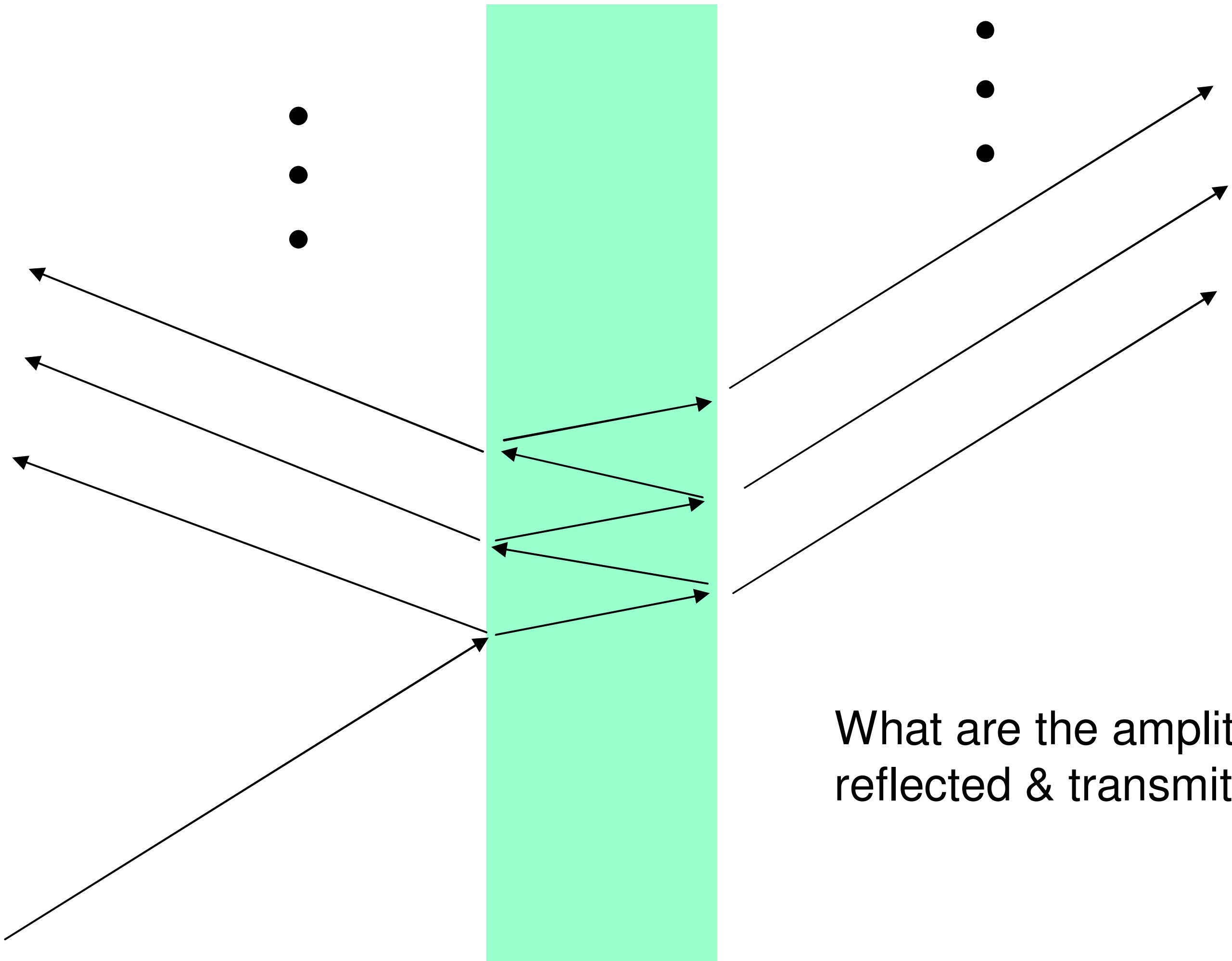


path diff. = $2t = (m + ½)\lambda_n = (m + ½)\lambda_o/n$

$m = 0, 1, 2, 3……$

min thickness (m = 0): $t = \lambda_o/4n$ (quarter-wave in coating) What if $n > n_{lens}$?

Multiple reflections



What are the amplitude of the reflected & transmitted waves?