Chapter 24

Capacitance & Dielectrics

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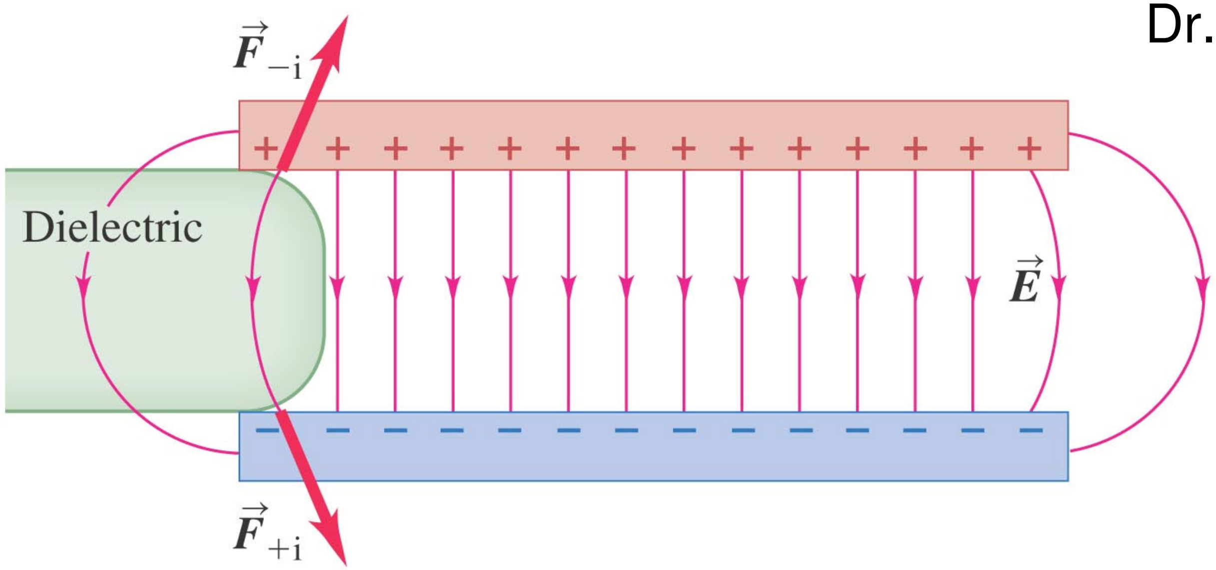
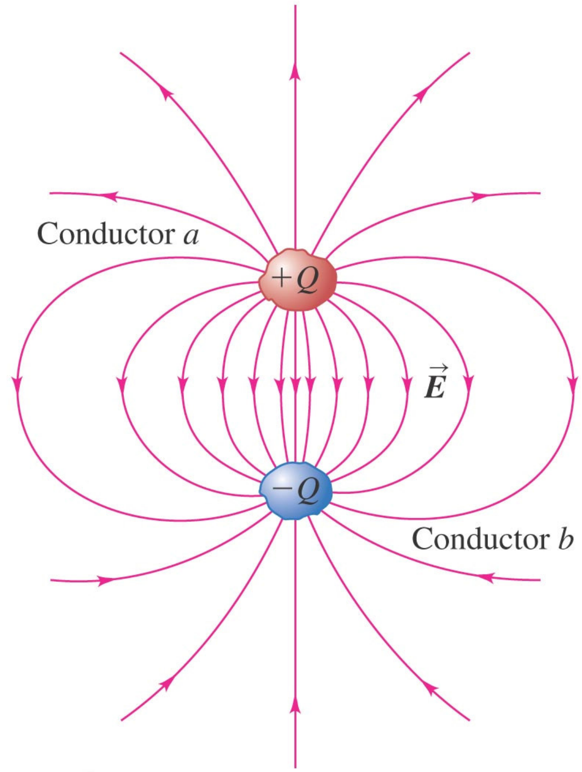


Goals for Chapter 24

- To consider capacitors and capacitance
- To study the use of capacitors in series and capacitors in parallel
- To determine the energy in a capacitor
- To examine dielectrics and see how different dielectrics lead to differences in capacitance

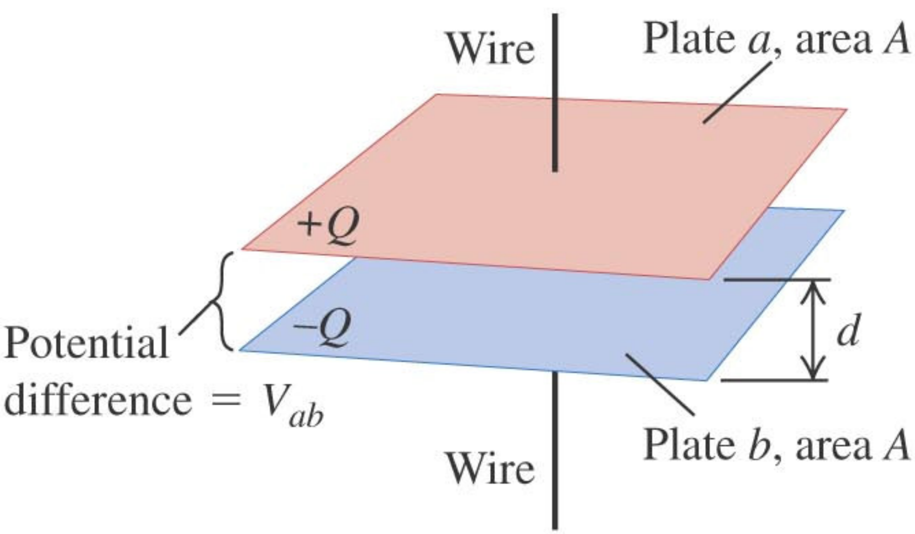
Keep charges apart and you get capacitance

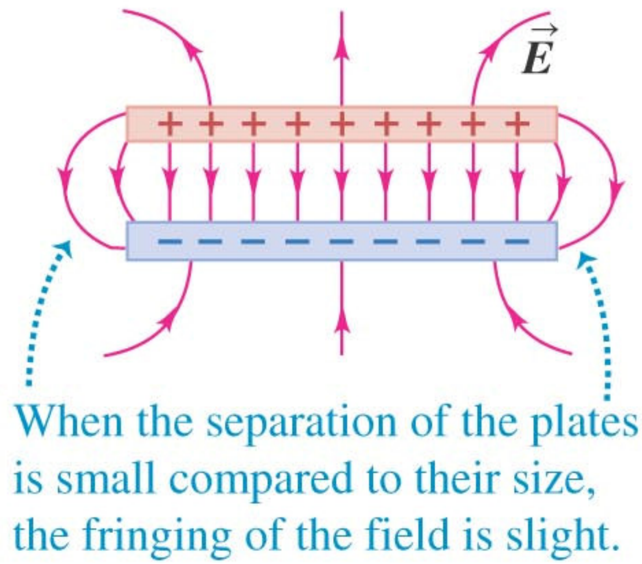
$$C \equiv \frac{Q}{V} = \frac{Q}{Ed} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_o} d} = \frac{\varepsilon_o A}{d}$$ parallel plates



(a) Arrangement of the capacitor plates

(b) Side view of the electric field $\vec{E}$



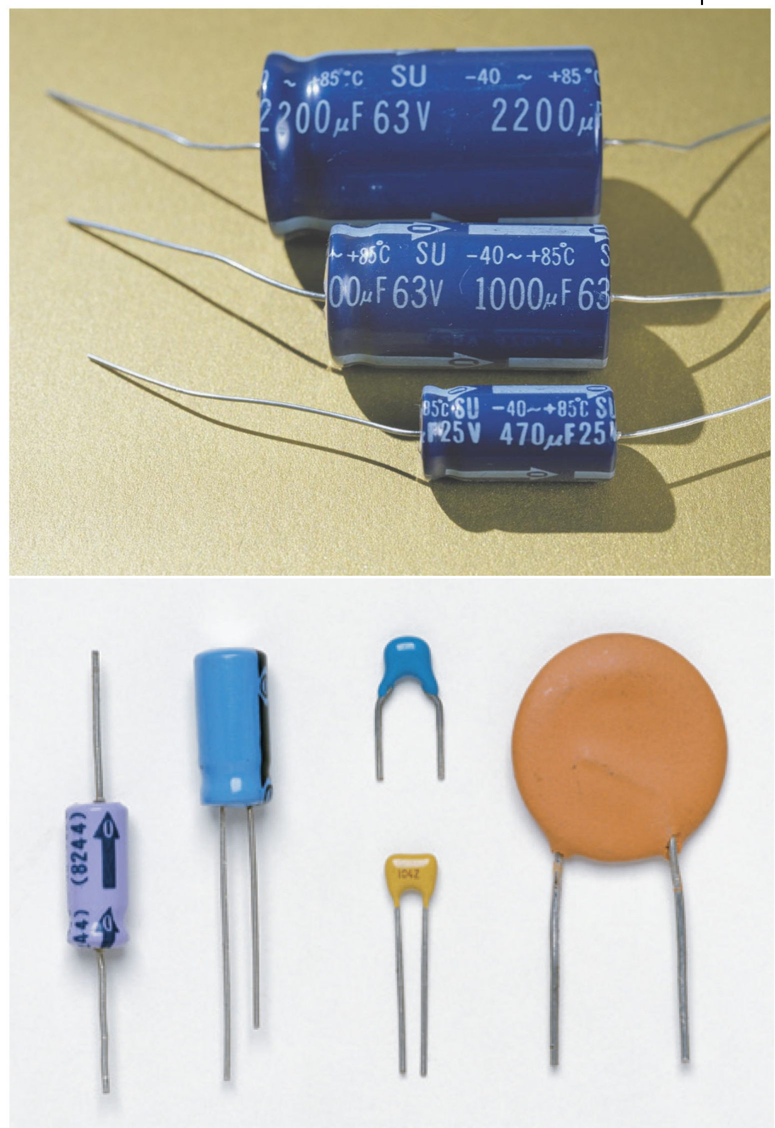


The unit of capacitance, the farad, is very large

- Commercial capacitors for home electronics are often cylindrical, from the size of a grain of rice to that of a large cigar.
- Capacitors like those mentioned above and pictured at right are microfarad capacitors.

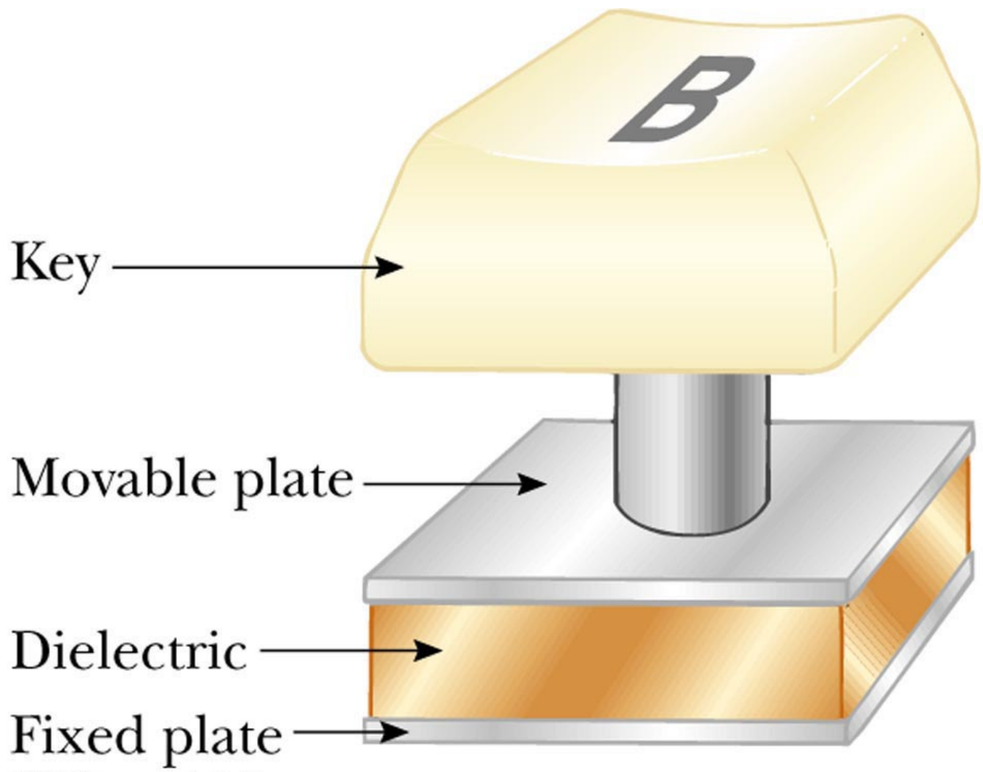
$$C \equiv \frac{Q}{V} = \frac{\varepsilon_o A}{d}$$

ε_o is a very small number.



Applications of Capacitors – e.g. Computers

- Computers use capacitors in many ways
  - Some keyboards use capacitors at the bases of the keys
  - When the key is pressed, the capacitor spacing decreases and the capacitance increases
  - The key is recognized by the change in capacitance



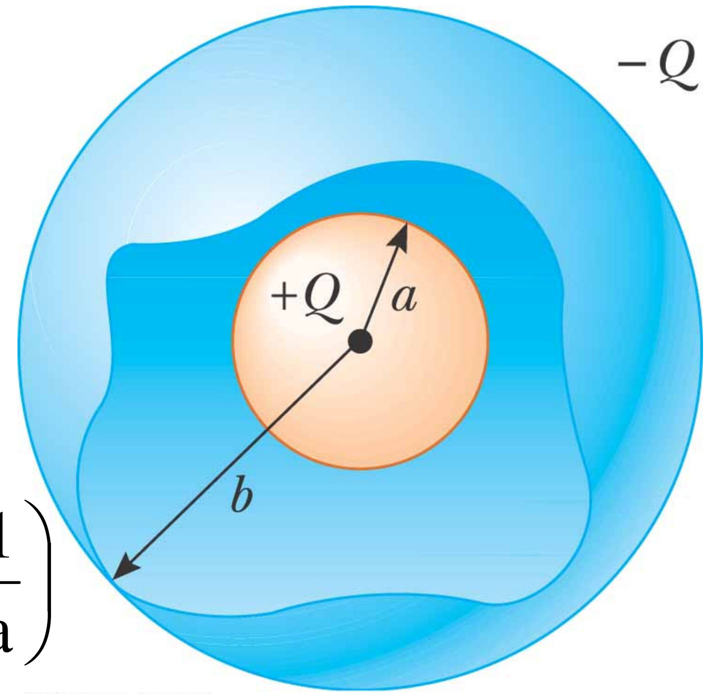
Capacitance of a Spherical Capacitor

$$E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

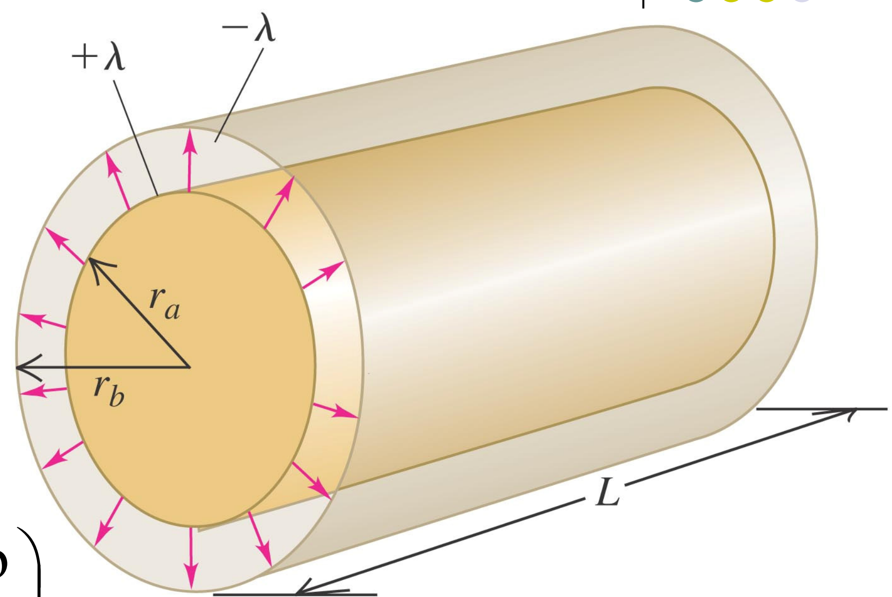
$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$V = -\int E \cdot d\ell = -\frac{Q}{4\pi\varepsilon_o}\int_a^b \frac{dr}{r^2} = \frac{Q}{2\pi\varepsilon_o}\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$C = \left|\frac{Q}{V}\right| = \frac{Q}{\frac{Q}{2\pi\varepsilon_o}\left(\frac{1}{b} - \frac{1}{a}\right)} = \frac{2\pi\varepsilon_o}{\left(\frac{1}{b} - \frac{1}{a}\right)}$$



Capacitance of a Cylindrical Capacitor



$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

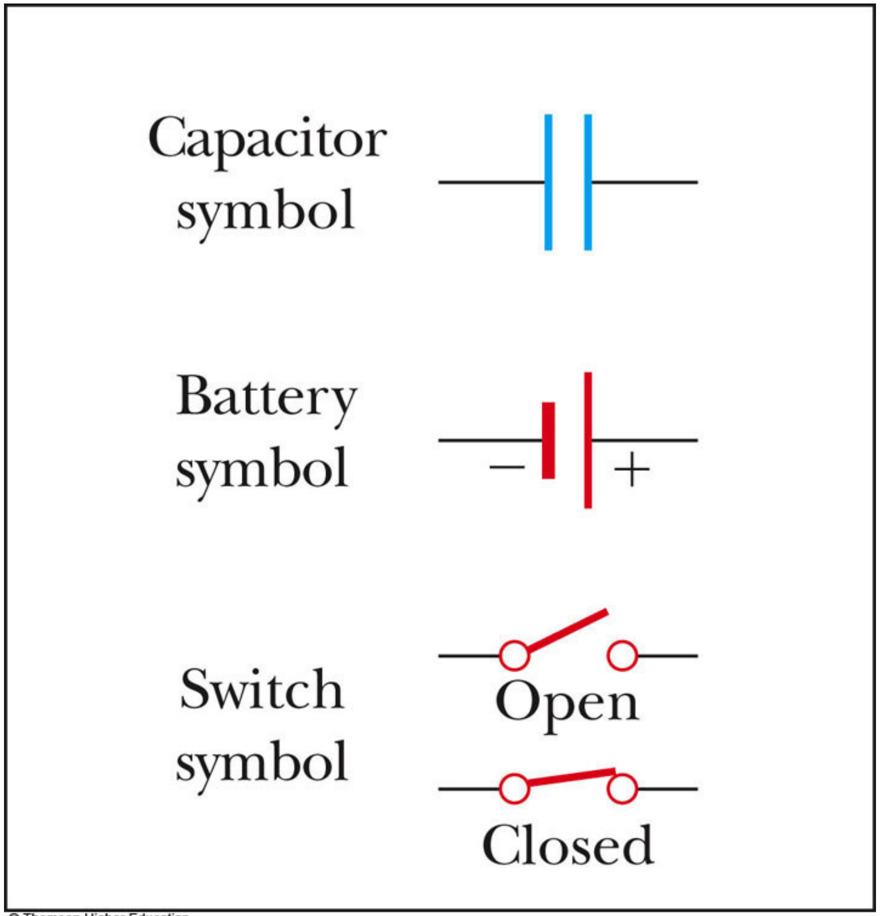
$$V = -\int \mathbf{E}\cdot d\ell = -\frac{\lambda}{2\pi\varepsilon_o}\int_a^b \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_o}\ln\left(\frac{b}{a}\right)$$

$$C = \left|\frac{Q}{V}\right| = \frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_o}\ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_o L}{\ln\left(\frac{b}{a}\right)}$$

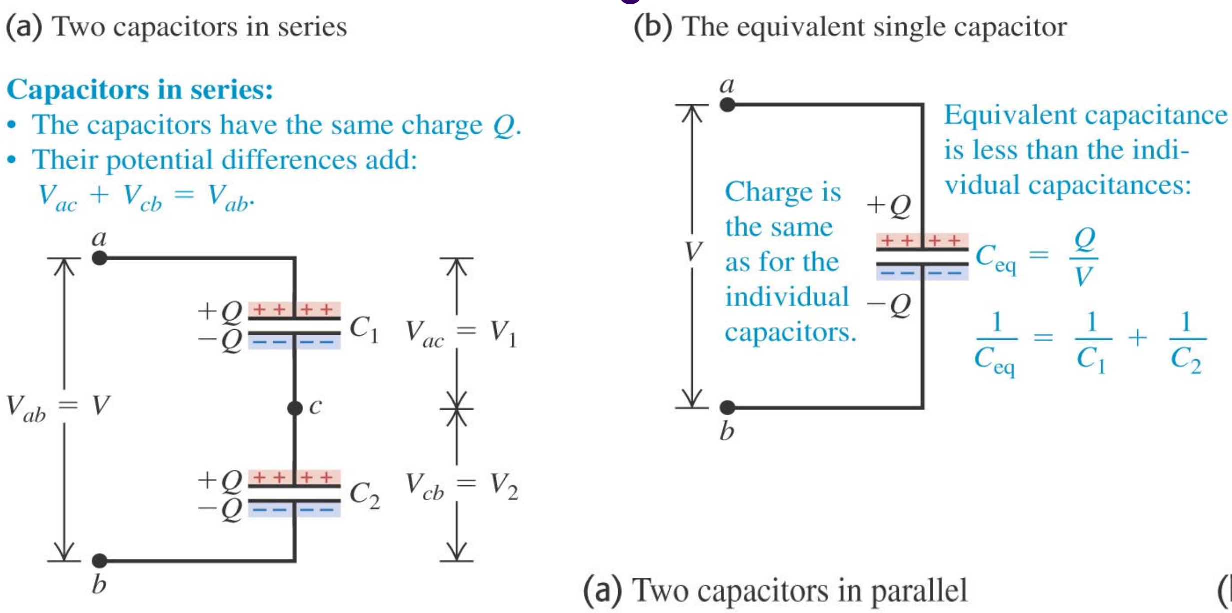
$$C' \equiv \frac{C}{L} = \frac{2\pi\varepsilon_o}{\ln\left(\frac{b}{a}\right)}$$

Circuit Symbols

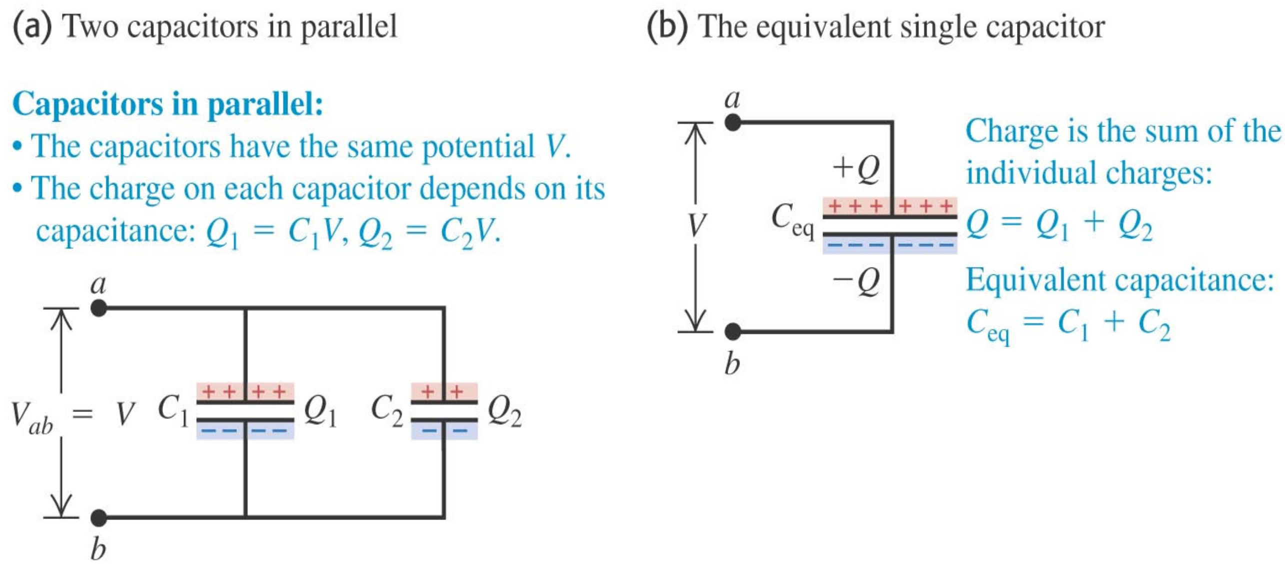
- A circuit diagram is a simplified representation of an actual circuit
- Circuit symbols are used to represent the various elements
- Lines are used to represent wires
- The battery's positive terminal is indicated by the longer line



Capacitors may be connected one or many at a time



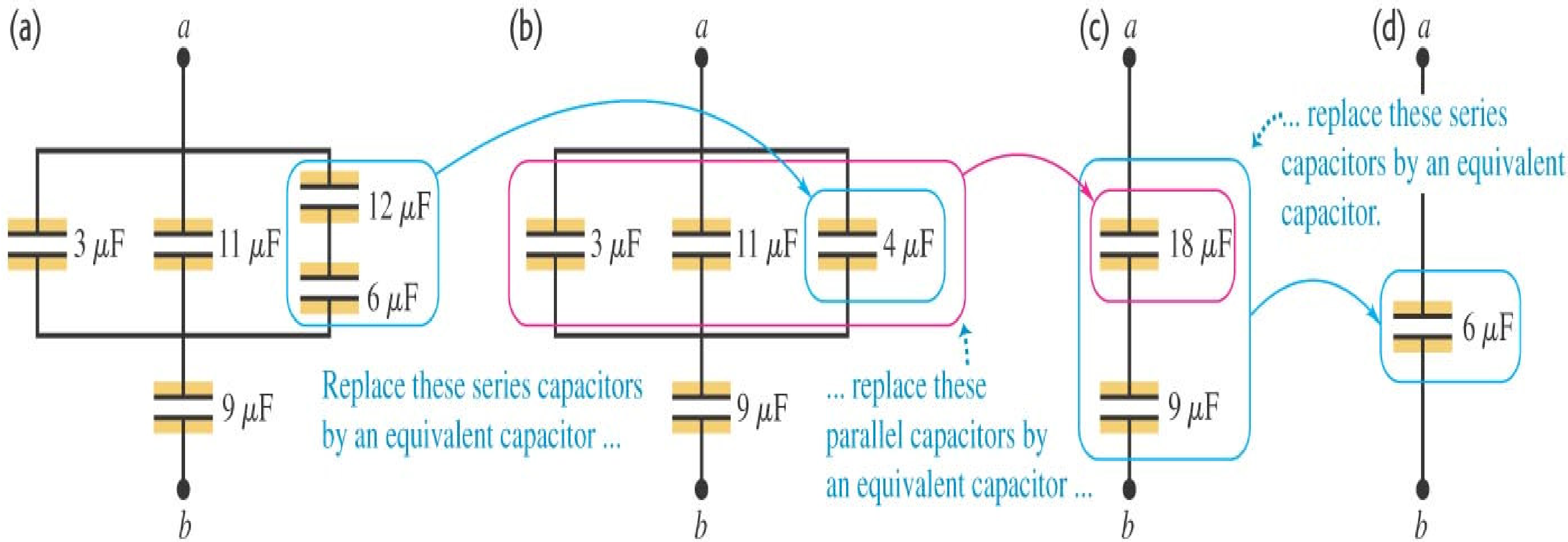
In series,
same Q.
$V = V_1 + V_2 + \ldots$



In parallel,
same V.
$Q = Q_1 + Q_2 + \ldots$

Calculations regarding capacitance

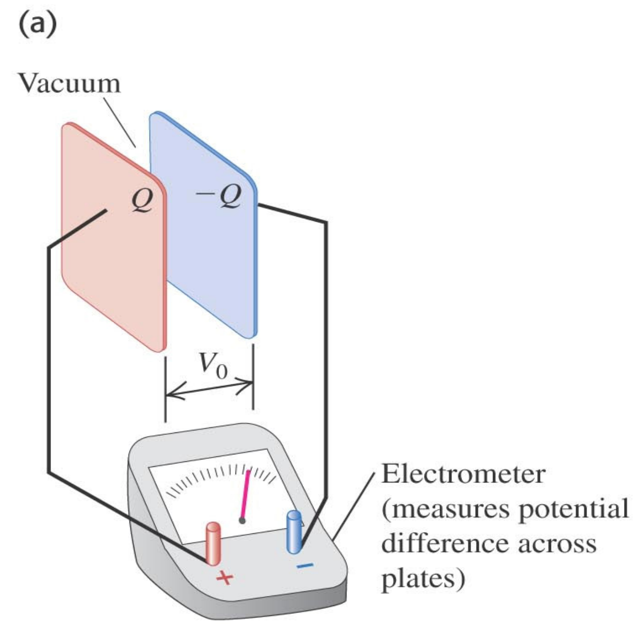
One step at a time !

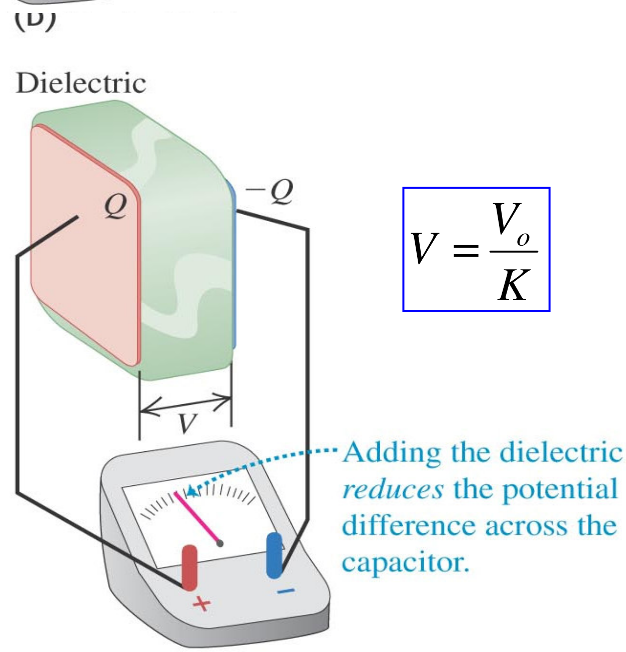


More examples…

Dielectrics change the potential difference

- The potential between to parallel plates of a capacitor changes when the material between the plates changes. It does not matter if the plates are rolled into a tube as they are in Figure 24.13 or if they are flat as shown in Figure 24.14.





$$V = \frac{V_o}{K}$$

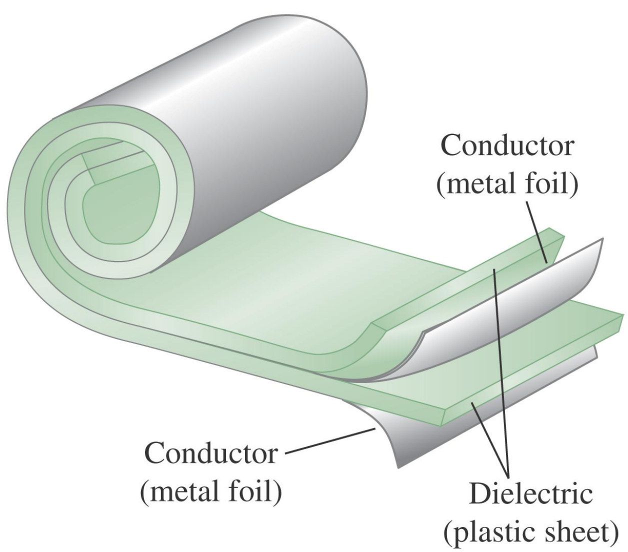


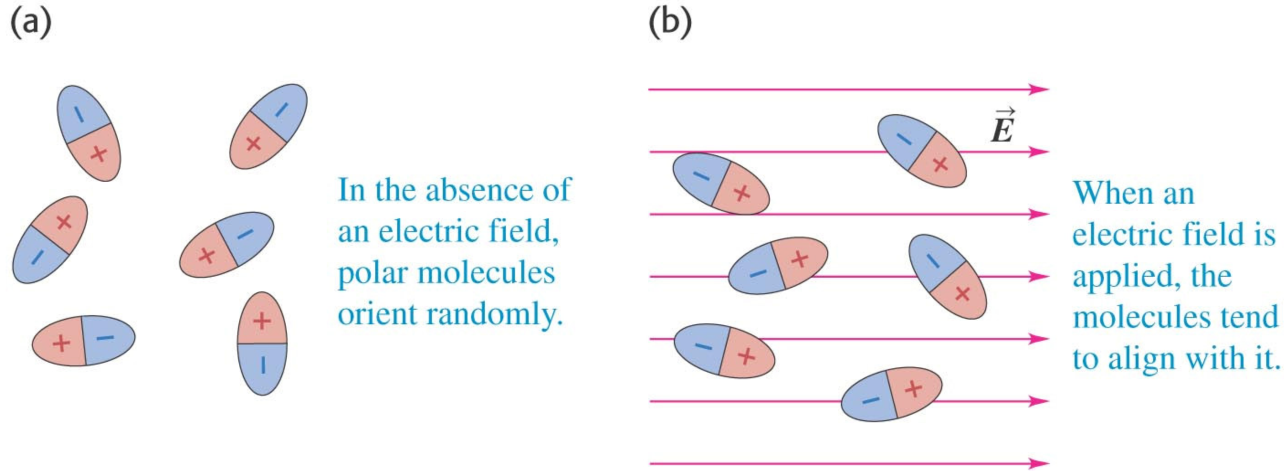
Table 24.1—Dielectric constants

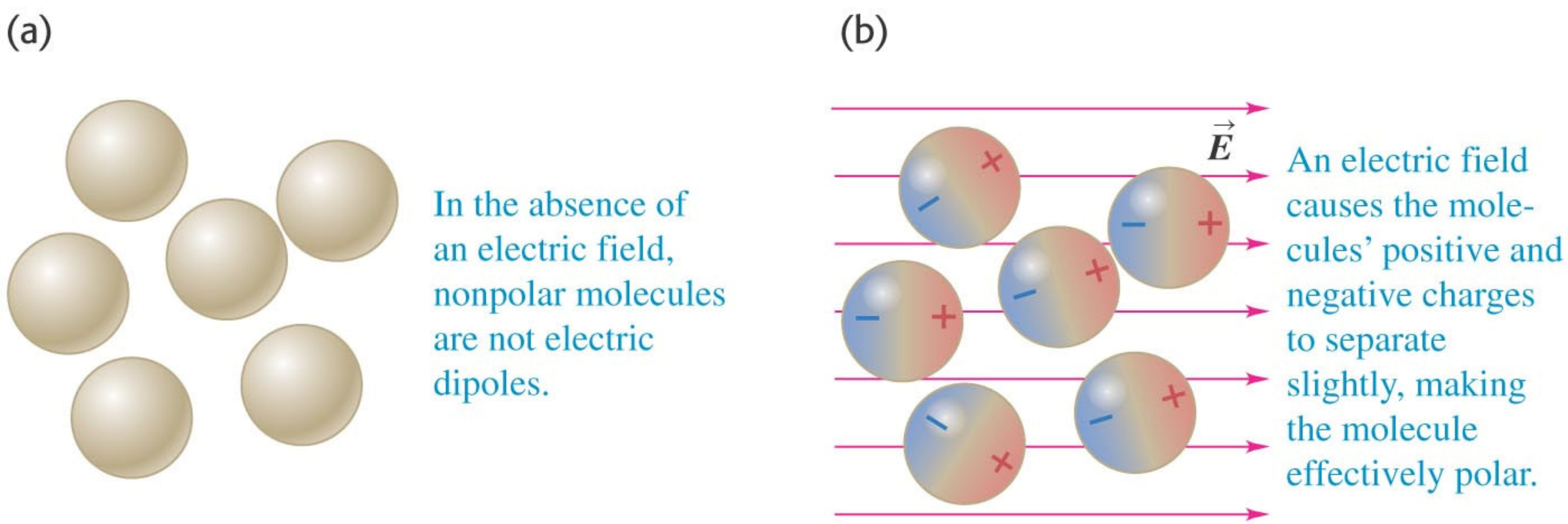
Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

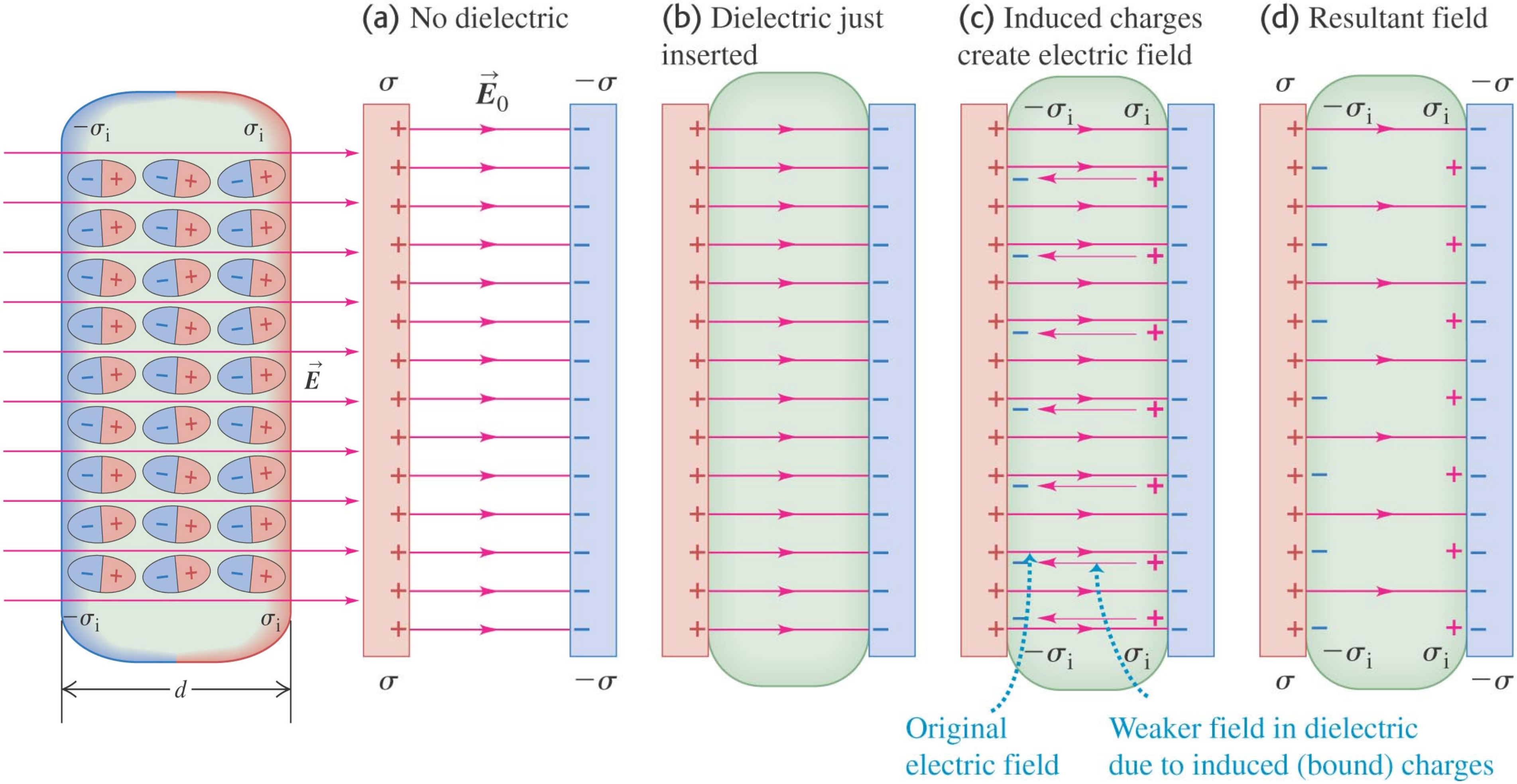
Molecular models

- Figure 24.18 (at right) and Figure 24.19 (below) show the effect of an applied field on individual molecules.





Polarization and electric field lines



Energy Stored in a Capacitor

- Assume the capacitor is being charged and, at some point, has a charge *q* on it
- The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$

- The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Energy, cont

- The work done in charging the capacitor appears as electric potential energy *U*:

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

$= \frac{1}{2} CV^2$

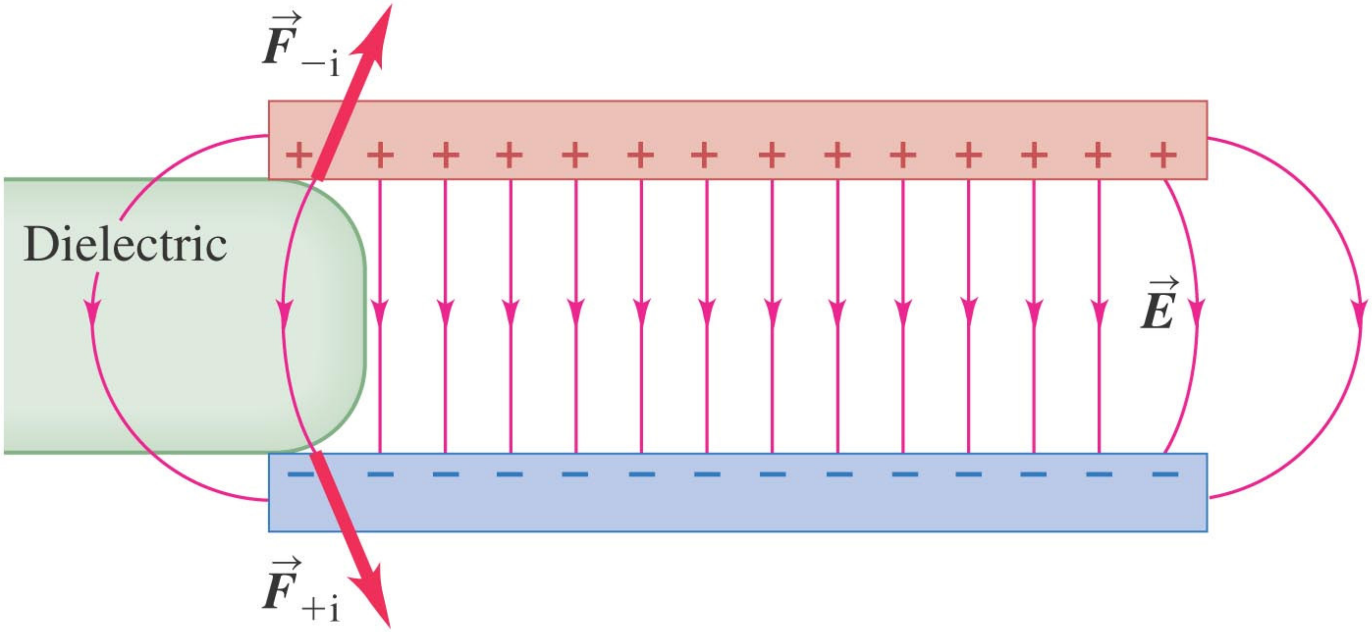
- This applies to a capacitor of any geometry
- The energy stored increases as the charge increases and as the potential difference increases
- In practice, there is a maximum voltage before discharge occurs between the plates

Energy, final

- The energy can be considered to be stored in the electric field
- For a parallel-plate capacitor, the energy can be expressed in terms of the field as $U = ½ (\varepsilon_o Ad)E^2$
- It can also be expressed in terms of the energy density (energy per unit volume)

  $u_E = ½\ \varepsilon_o E^2$

Examples to consider, capacitors with and without dielectrics



Electric energy density $= \frac{1}{2}\varepsilon E^2 = \frac{1}{2}k\varepsilon_o E^2$
(Joules/m^3)

Dielectric breakdown

- A very strong electrical field can exceed the strength of the dielectric to contain it. Table 24.2 at the bottom of the page lists some limits.

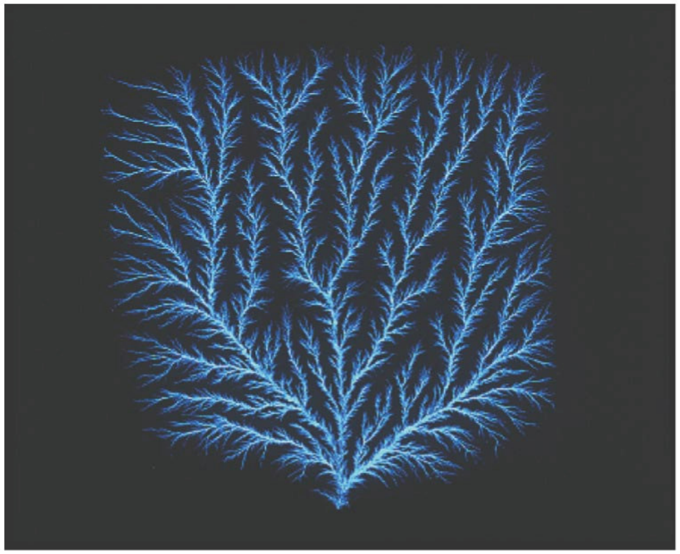


Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

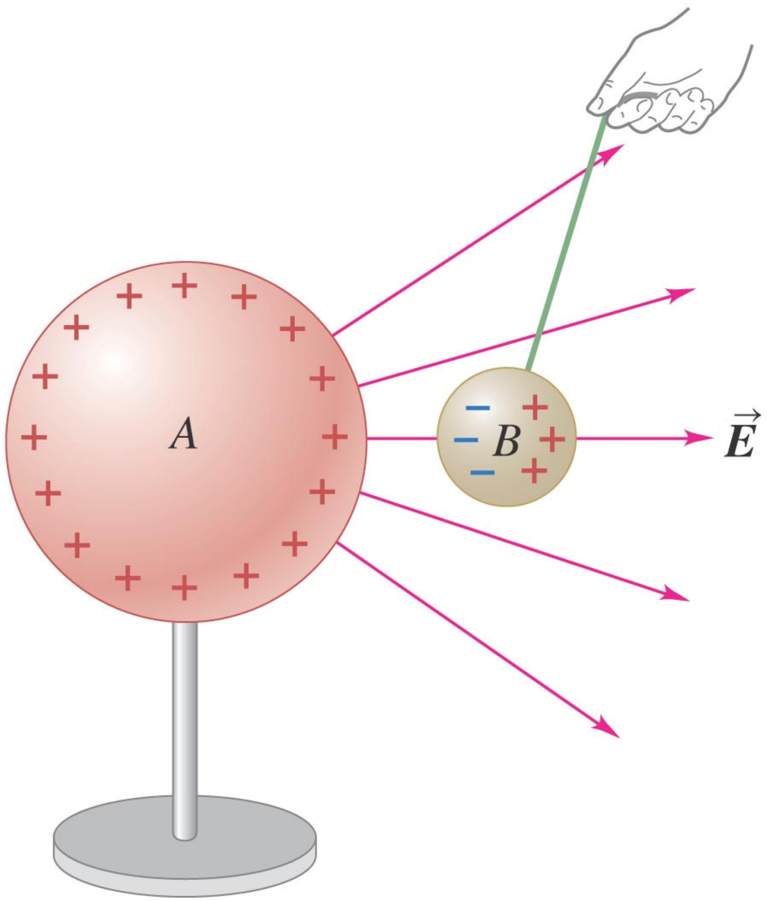
TABLE 26.1

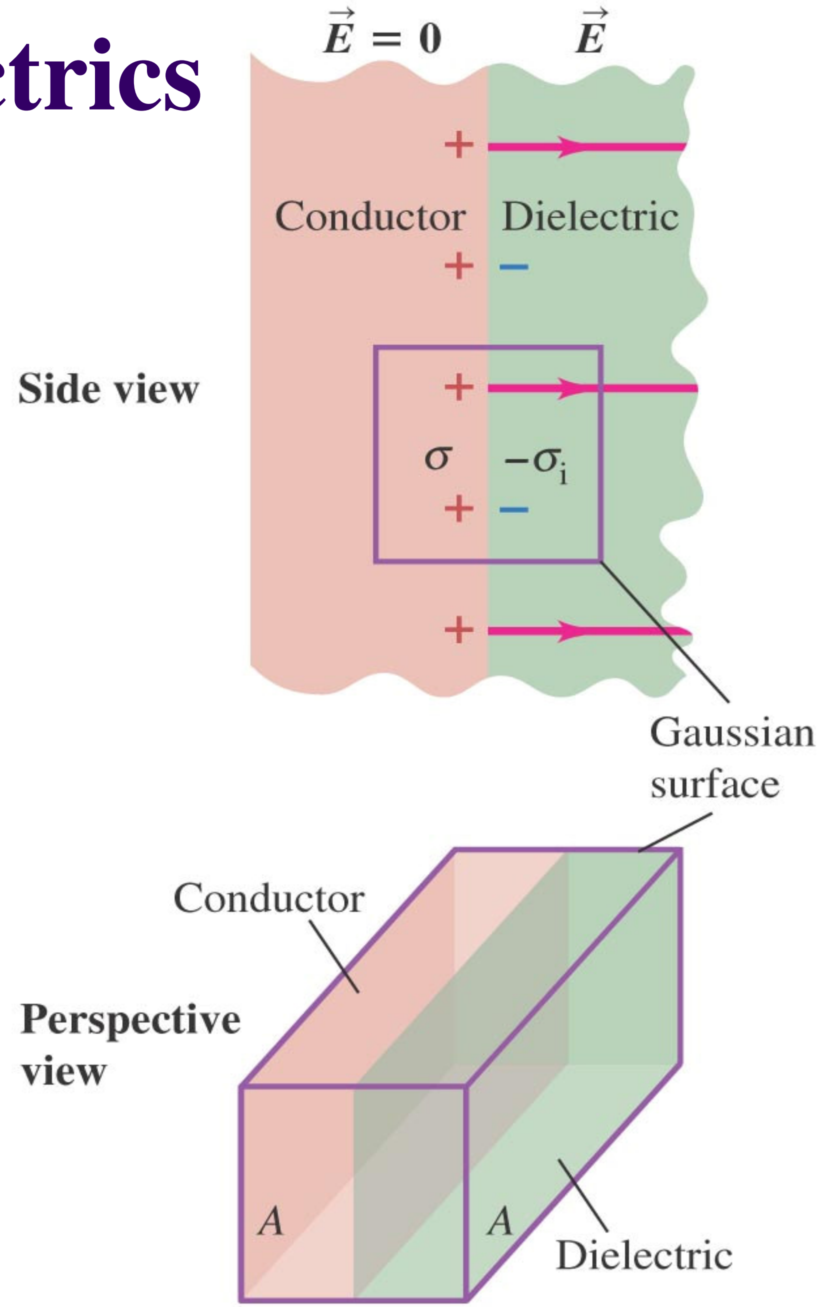
Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

Gauss's Law in dielectrics





$$EA_{Gauss} = \frac{Q_{total,inside}}{\varepsilon_o} \approx \frac{Q_{free,inside}}{K\varepsilon_o}$$