Resultants of Three Dimensional Forces Example Problem

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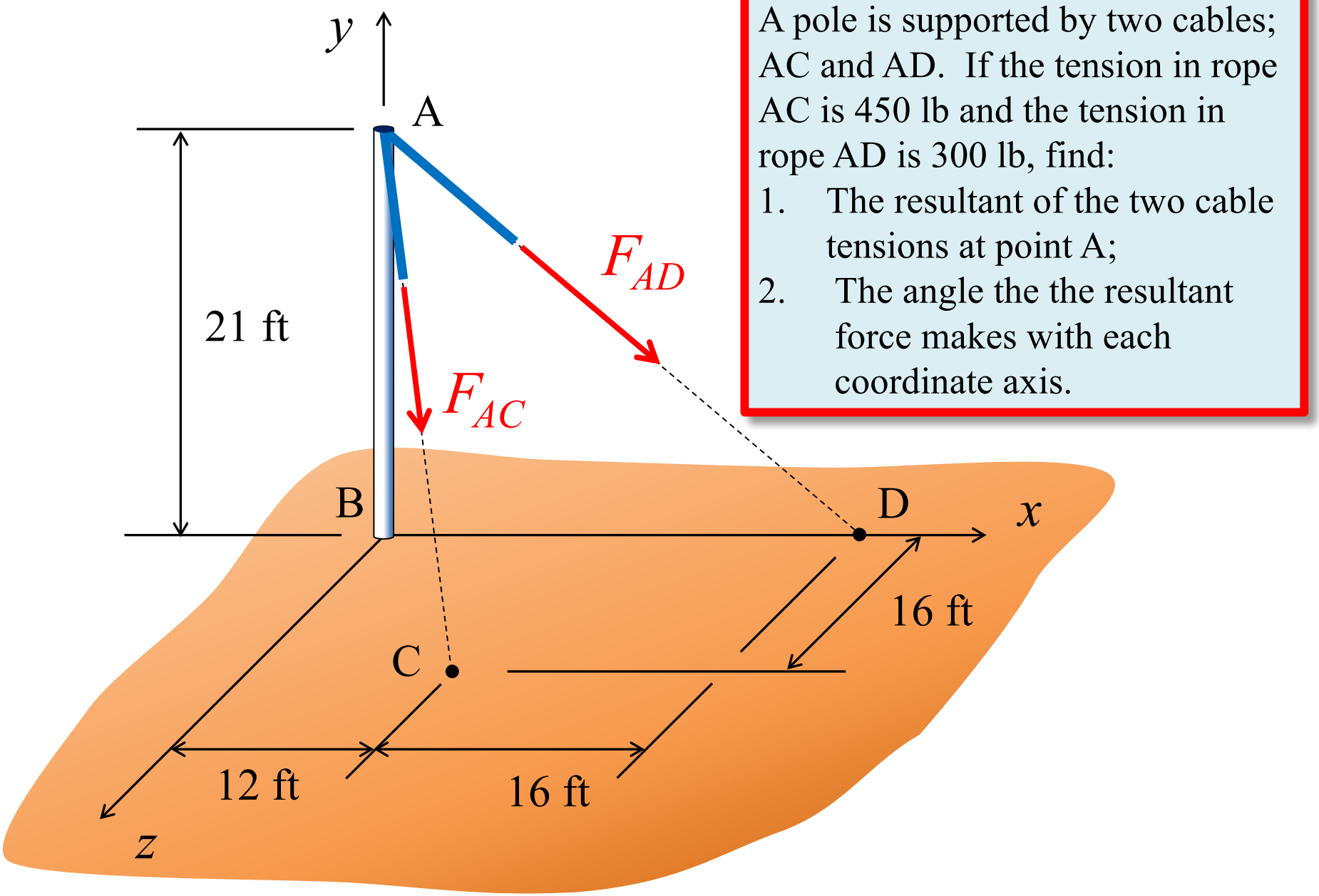
A pole is supported by two cables; AC and AD. If the tension in rope AC is 450 lb and the tension in rope AD is 300 lb, find:

1. The resultant of the two cable tensions at point A;
2. The angle the the resultant force makes with each coordinate axis.



Express Force F_{AC} in Cartesian Vector Form

Coordinates of Point C: (12, 0, 16)

Coordinates of Point A: (0, 21, 0)

$$d_x = x_C - x_A = 12 - 0 = 12 \text{ ft}$$

$$d_y = y_C - y_A = 0 - 21 = -21 \text{ ft}$$

$$d_z = z_C - z_A = 16 - 0 = 16 \text{ ft}$$

Tip minus Tail

$$d = \sqrt{(12)^2 + (-21)^2 + (16)^2} = 29 \text{ ft}$$

$$F_{ACx} = \left(\frac{(x_C - x_A)}{d}\right)(F_{AC}) = \left(\frac{12 \text{ ft}}{29 \text{ ft}}\right)(450 \text{ lb}) = 186.21 \text{ lb}$$

$$F_{ACy} = \left(\frac{(y_C - y_A)}{d}\right)(F_{AC}) = \left(\frac{-21 \text{ ft}}{29 \text{ ft}}\right)(450 \text{ lb}) = -325.86 \text{ lb}$$

$$F_{ACz} = \left(\frac{(z_C - z_A)}{d}\right)(F_{AC}) = \left(\frac{16 \text{ ft}}{29 \text{ ft}}\right)(450 \text{ lb}) = 248.28 \text{ lb}$$

$$\boldsymbol{F_{AC}} = 186.21\hat{\imath} - 325.86\hat{\jmath} + 248.28\hat{k} \text{ lb}$$

Express Force F_{AD} in Cartesian Vector Form

Coordinates of Point D: (28, 0, 0)

Coordinates of Point A: (0, 21, 0)

$$d_x = x_D - x_A = 28 - 0 = 28 \text{ ft}$$

$$d_y = y_D - y_A = 0 - 21 = -21 \text{ ft}$$

$$d_z = z_D - z_A = 0 - 0 = 0$$

Tip minus Tail

$$d = \sqrt{(28)^2 + (-21)^2 + (0)^2} = 35 \text{ ft}$$

$$F_{ADx} = \left(\frac{(x_D - x_A)}{d}\right)(F_{AD}) = \left(\frac{28 \text{ ft}}{35 \text{ ft}}\right)(300 \text{ lb}) = 240 \text{ lb}$$

$$F_{ADy} = \left(\frac{(y_D - y_A)}{d}\right)(F_{AD}) = \left(\frac{-21 \text{ ft}}{35 \text{ ft}}\right)(300 \text{ lb}) = -180 \text{ lb}$$

$$F_{ADz} = \left(\frac{(z_D - z_A)}{d}\right)(F_{AD}) = \left(\frac{0}{35 \text{ ft}}\right)(300 \text{ lb}) = 0$$

$$\boldsymbol{F_{AD}} = 240\hat{\imath} - 180\hat{\jmath} \text{ lb}$$

Resultant in Cartesian Vector Form

$$\boldsymbol{R} = \boldsymbol{F_{AC}} + \boldsymbol{F_{AD}}$$

$$\boldsymbol{R} = \left(\sum F_x\right)\hat{\imath} + \left(\sum F_y\right)\hat{\jmath} + \left(\sum F_y\right)\hat{k}$$

$$\boldsymbol{F_{AC}} = 186.21\hat{\imath} - 325.86\hat{\jmath} + 248.28\hat{k} \text{ lb}$$

$$\boldsymbol{F_{AD}} = 240\hat{\imath} - 180\hat{\jmath} \text{ lb}$$

$$\boldsymbol{R} = 426.21\hat{\imath} - 505.86\hat{\jmath} + 248.28\hat{k} \text{ lb}$$

Total Magnitude and Angles

$$R = \sqrt{(R_x)^2 + \left(R_y\right)^2 + (R_z)^2}$$

$$R = \sqrt{(426.21)^2 + (-505.86)^2 + (248.28)^2} = 706.53 \text{ lb}$$

$$\cos\theta_x = \frac{R_x}{R} = \frac{426.21 \text{ lb}}{706.53 \text{ lb}} = 0.6032 \qquad \theta_x = \cos^{-1}(0.6032) = 52.9°$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{-505.86 \text{ lb}}{706.53 \text{ lb}} = -0.7160 \qquad \theta_y = \cos^{-1}(-0.7160) = 135.7°$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{248.28 \text{ lb}}{706.53 \text{ lb}} = 0.3514 \qquad \theta_z = \cos^{-1}(0.3514) = 69.4°$$

Resultant Force at Point A