Moment of a Force About a Point
Planar Example Problem

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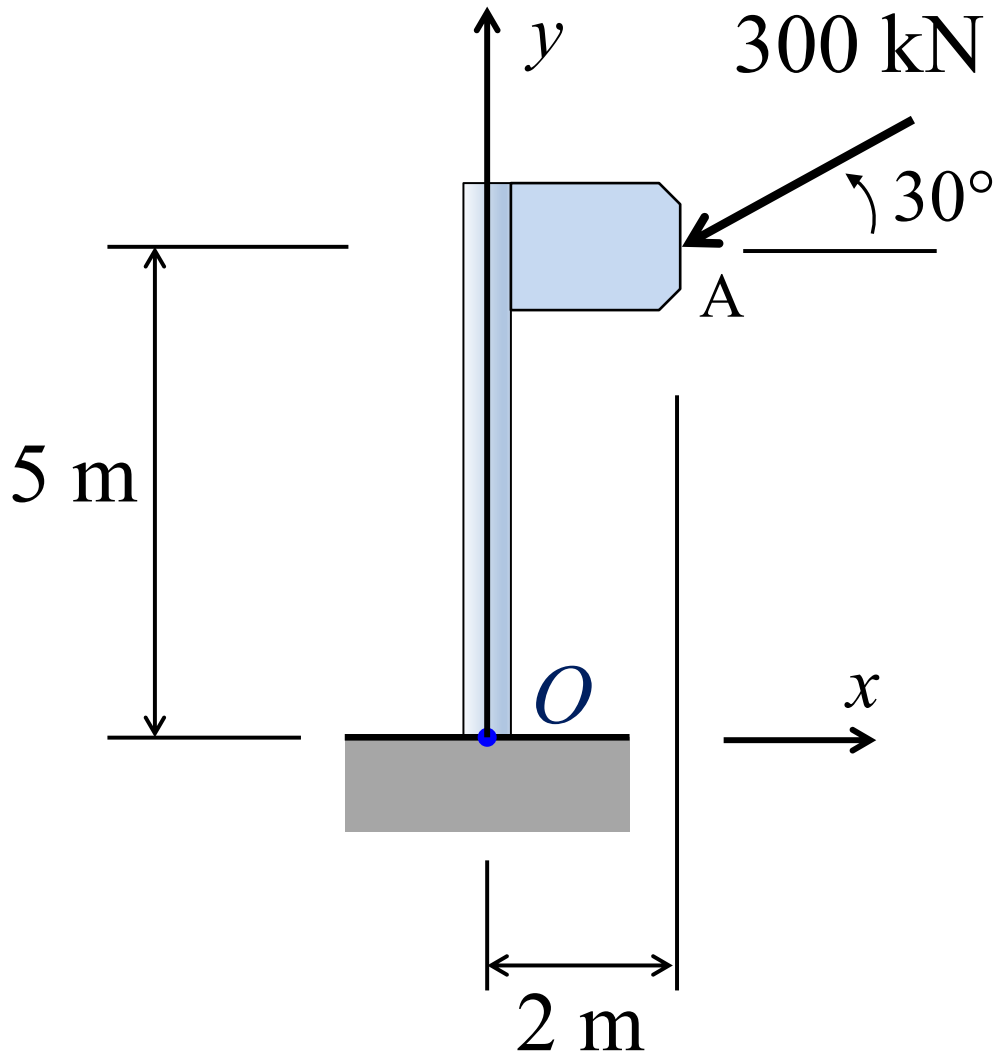
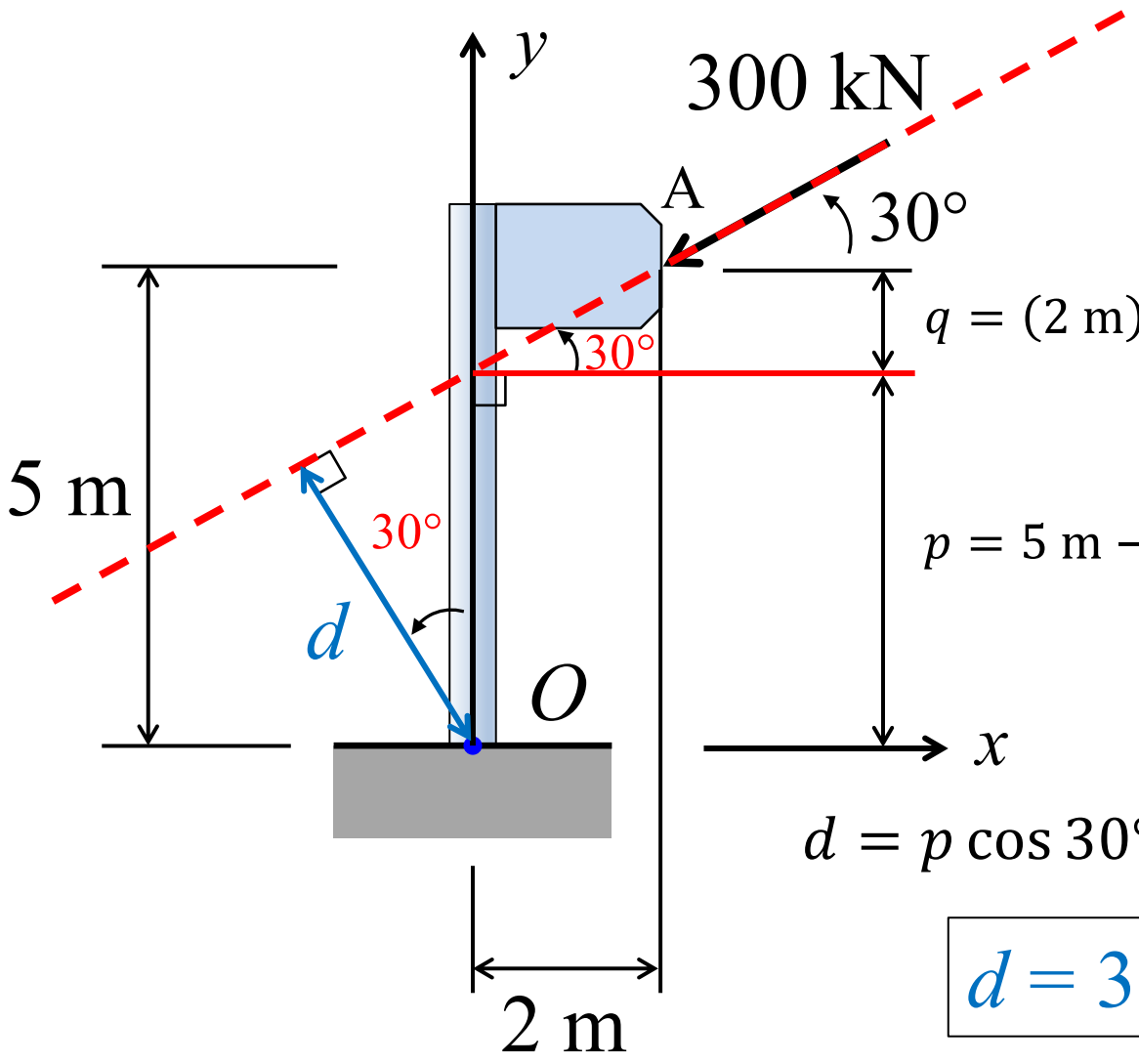
Example Problem



Find the moment of the force, applied at point A, about point ***O*** three different ways:

1. Using the perpendicular distance;
2. Adding the moment of each component;
3. Using the cross product of the position vector and the force vector.

Find the Perpendicular Distance to the Line of Action of the Force



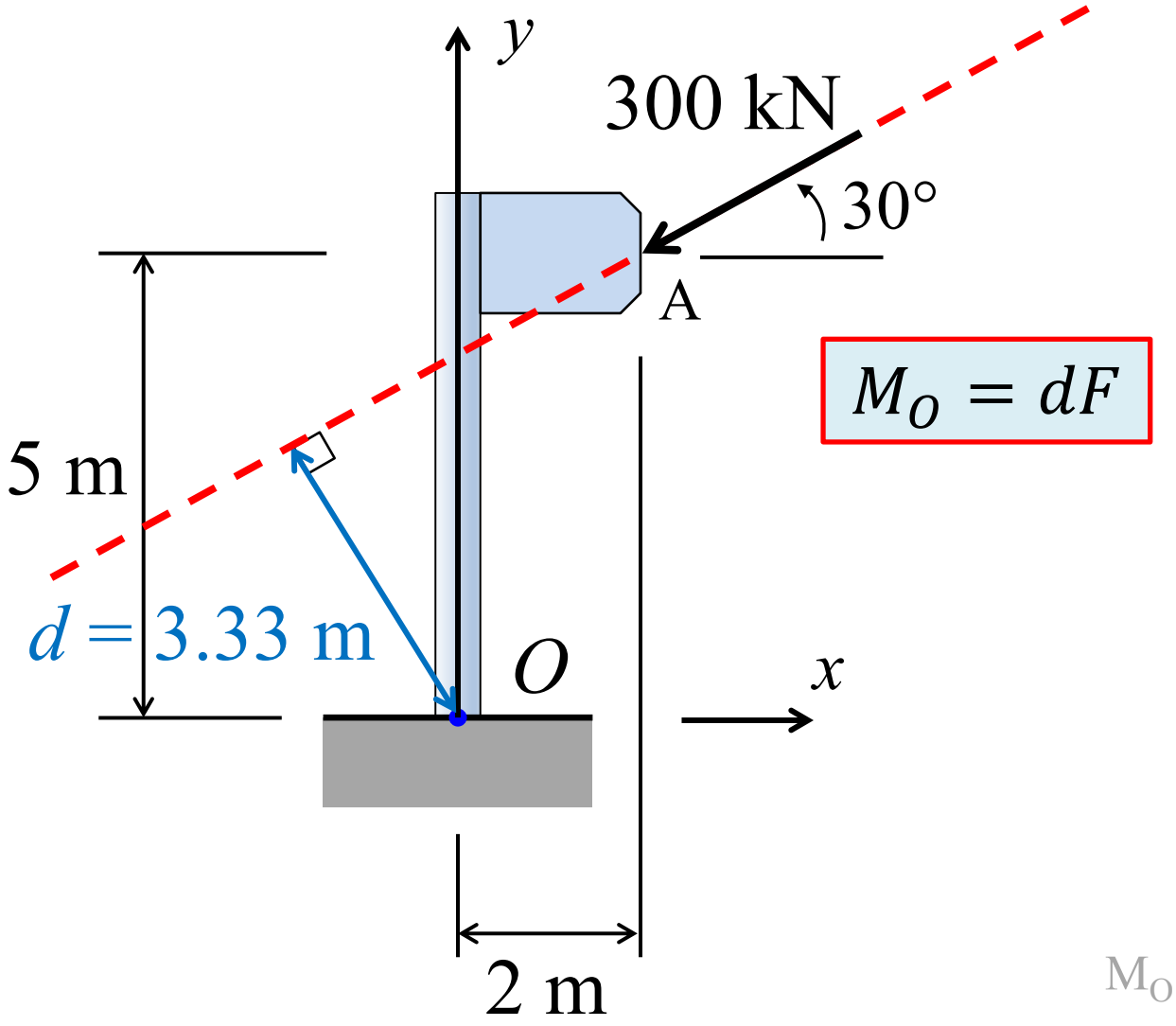
$q = (2\text{ m})\tan 30° = 1.1547\text{ m}$

$p = 5\text{ m} - q = 3.8453\text{ m}$

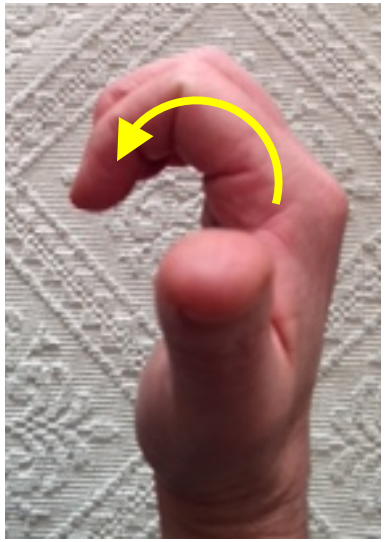
$d = p\cos 30° = (3.8453\text{ m})(0.866)$

$d = 3.33\text{ m}$

Find the Moment of the Force about Point *O* Using the Perpendicular Distance

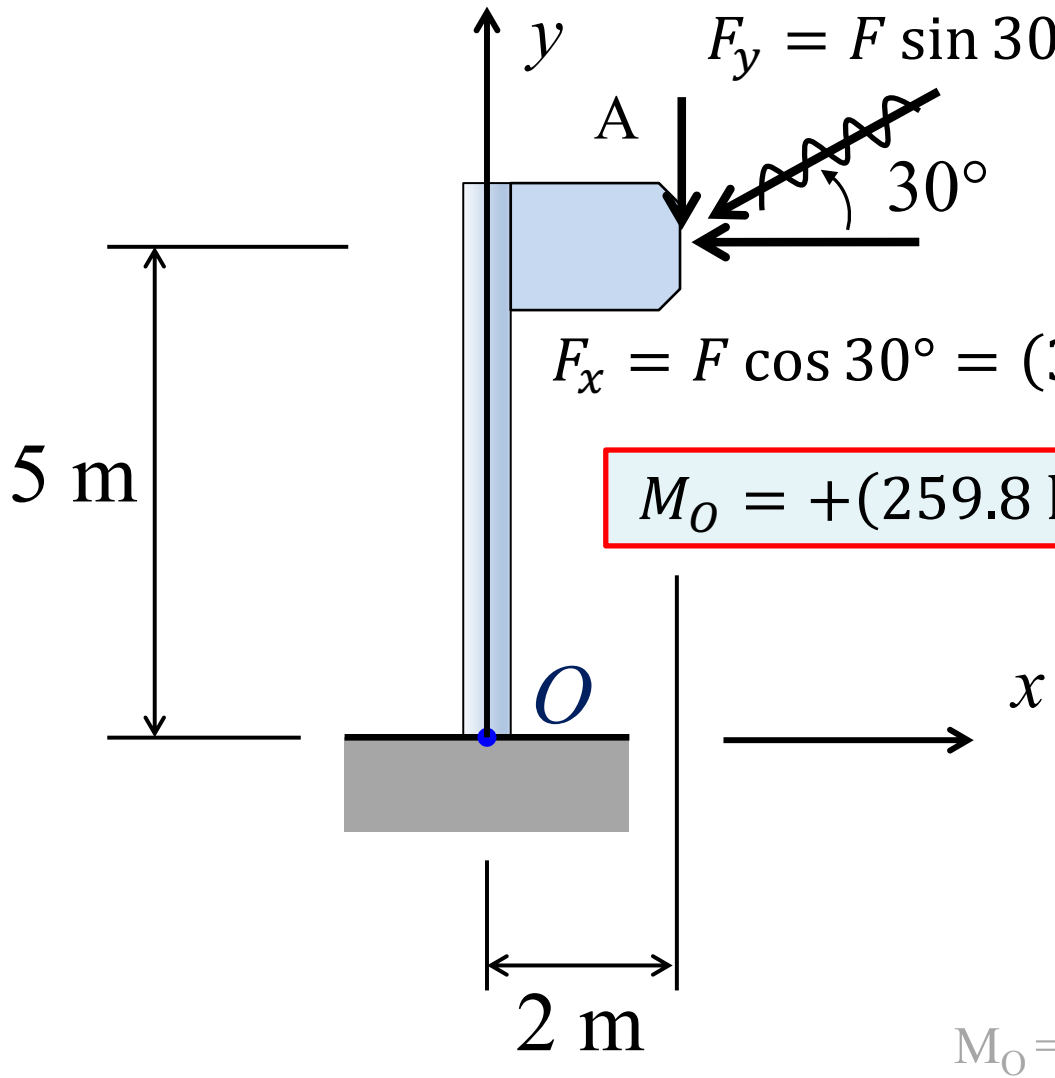


$$M_O = dF$$



$M_O = 999$ kN-m CCW

Sum the Moments of Each Component of the Force About Point *O*



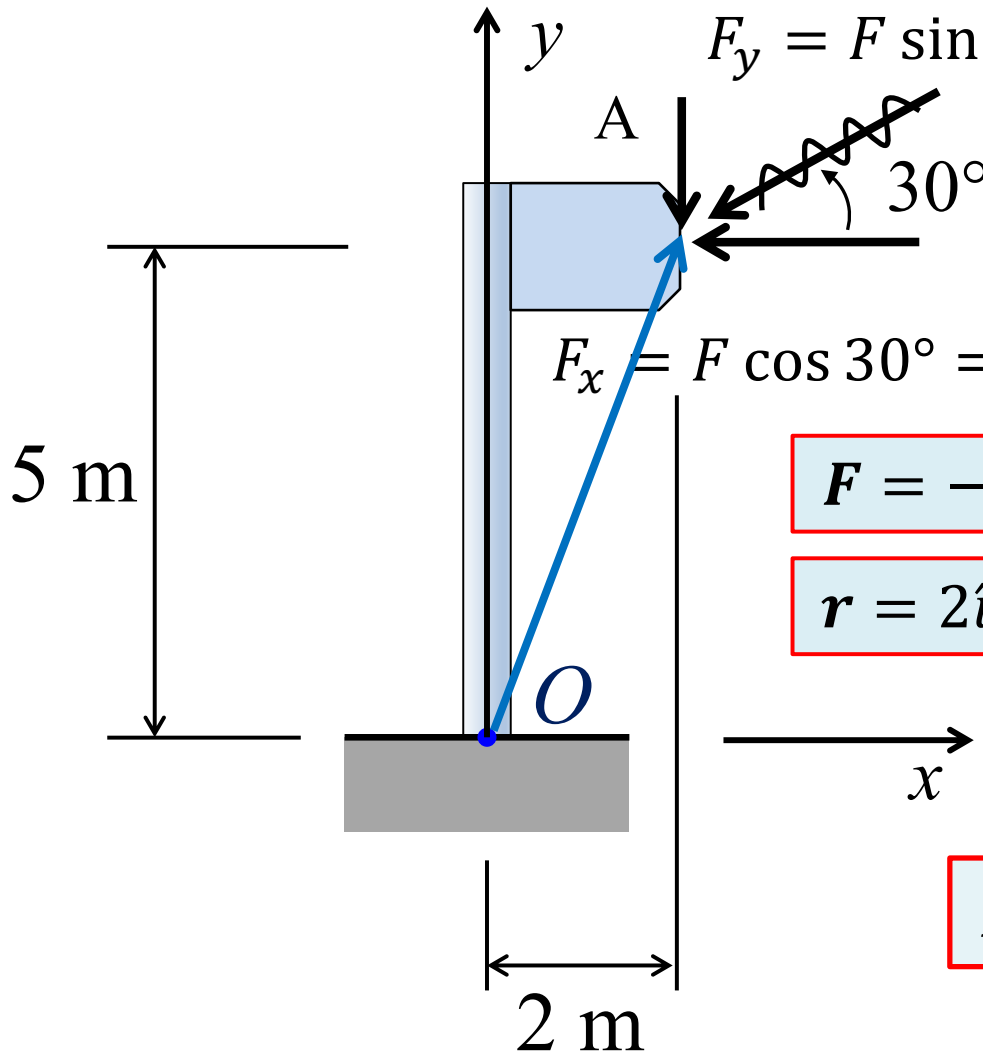
$F_y = F \sin 30° = (300\ \text{kN})(0.5) = \mathbf{150\ kN}$

$F_x = F \cos 30° = (300\ \text{kN})(0.866) = \mathbf{259.8\ kN}$

$M_O = +(259.8\ \text{kN})(5\ \text{m}) - (150\ \text{kN})(2\ \text{m})$

$M_O = 999$ kN-m CCW

Cross Product of the Position Vector and the Force Vector and in Cartesian Vector Form



$F_y = F \sin 30° = (300\ \text{kN})(0.5) = \mathbf{150\ kN}$

$F_x = F \cos 30° = (300\ \text{kN})(0.866) = \mathbf{259.8\ kN}$

$\boldsymbol{F} = -259.8\hat{\imath} - 150\hat{\jmath}\ \text{kN}$

$\boldsymbol{r} = 2\hat{\imath} + 5\hat{\jmath}\ \text{m}$

$\boldsymbol{M_O = r \times F}$

Cross Product Calculation in Matrix Form

$\boldsymbol{r} = 2\hat{\imath} + 5\hat{\jmath}\ \text{m}$

$\boldsymbol{F} = -259.8\hat{\imath} - 150\hat{\jmath}\ \text{kN}$

$$\boldsymbol{M_O} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 5 & 0 \\ -259.8 & -150 & 0 \end{vmatrix}$$

$$\boldsymbol{M_O} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 5 & 0 \\ -259.8 & -150 & 0 \end{vmatrix} \begin{matrix} \hat{\imath} & \hat{\jmath} \\ 2 & 5 \\ -259.8 & -150 \end{matrix}$$

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$$\boldsymbol{M_O} = [(2)(-150) - (5)(-259.8)]\hat{k}$$

$$\boldsymbol{M_O} = 999\hat{k}\ \text{kN-m}$$

$M_O = 999$ kN-m CCW