Equilibrium of Rigid Bodies and Construction Free-Body Diagrams

Steven Vukazich
San Jose State University

General procedure for the Analysis of Bodies in Static Equilibrium

- Choose the free body to isolate;
- Draw a **Free Body Diagram (FBD)** of the body;
  - Isolate the body from all of its surroundings,
  - Magnitudes and directions of all known and unknown forces acting on the body should be included and clearly indicated,
  - Indicate dimensions on the FBD,
- Write the **equations of equilibrium** and solve the equations for the unknown quantities.

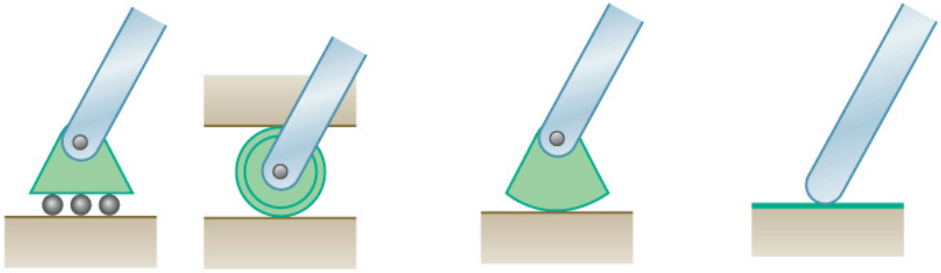
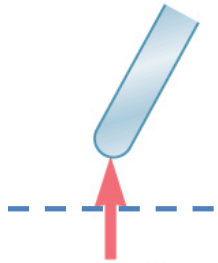
General Procedure for the Construction of Free Body Diagrams

- Choose the free body to isolate;
- Isolate the body from all of its surroundings;
- Magnitudes and directions of all known and unknown forces acting on the body should be included and clearly indicated;
- Dimensions should be indicated on the FBD.

Most errors in mechanics problems result from a mistake in the FBD

Reactive Forces at Supports for Planar Structures

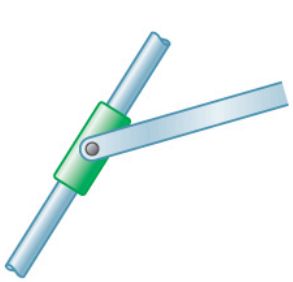
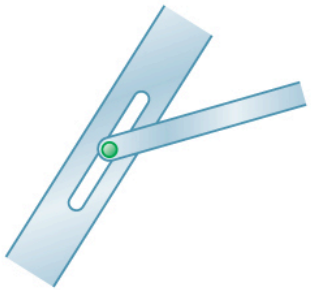
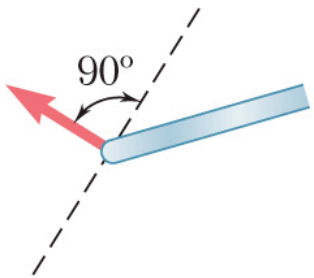
Reactive force with unknown magnitude and known line of action

Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1

force is perpendicular to surface

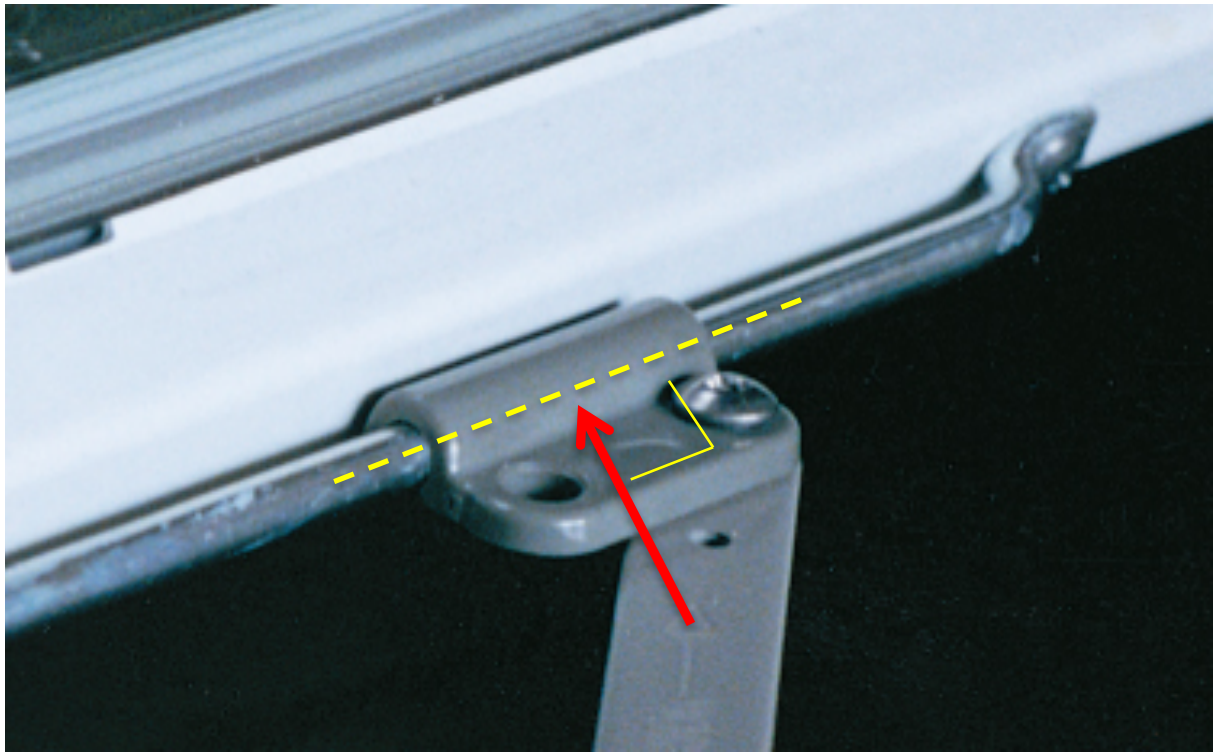
Example of a Roller Support

Reactive force with unknown magnitude and known line of action

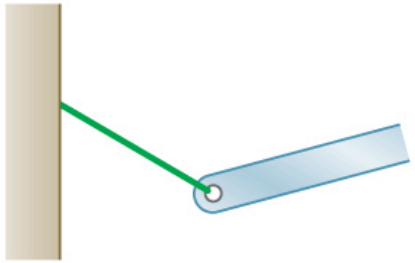
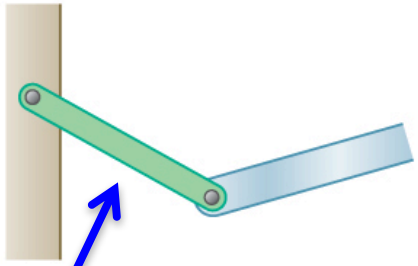
Support or Connection	Reaction	Number of Unknowns
 Collar on frictionless rod  Frictionless pin in slot	 Force with known line of action	1

Force is perpendicular to rod or slot

Example of a collar on a guide rod



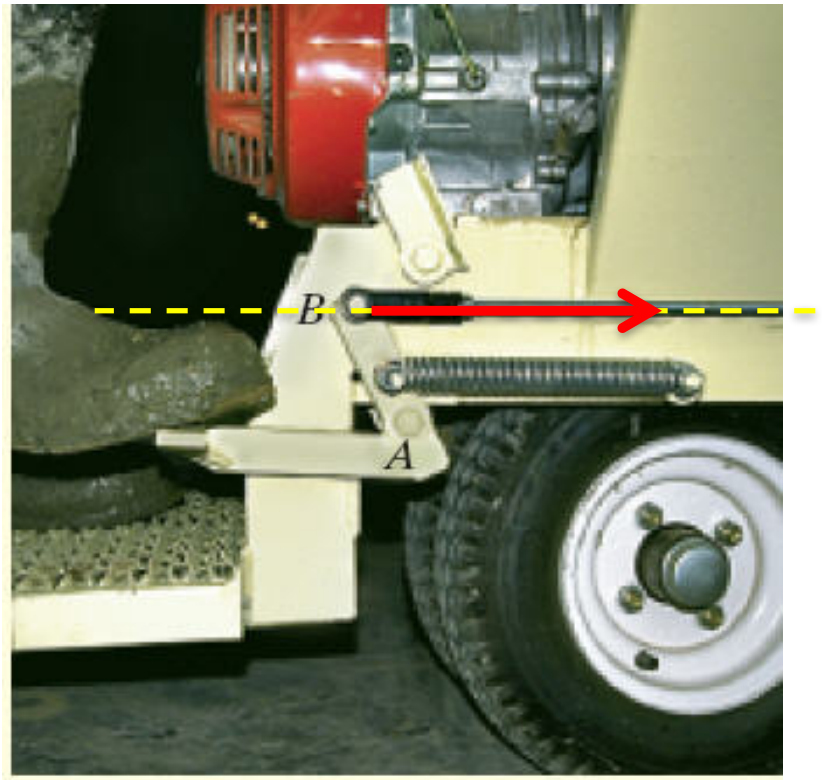
Reactive force with unknown magnitude and known line of action

Support or Connection	Reaction	Number of Unknowns
 Short cable  Short link	 Force with known line of action	1

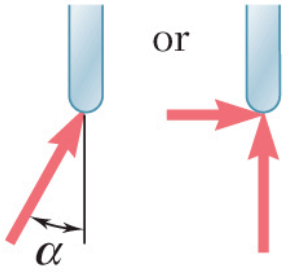
Link is a two-force member

Force is directed along the line of the cable or link

Examples of short links

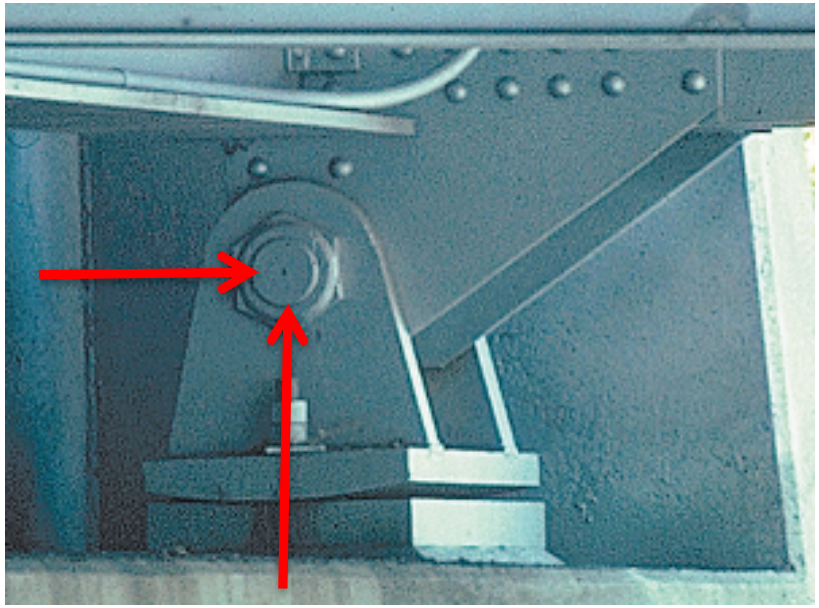


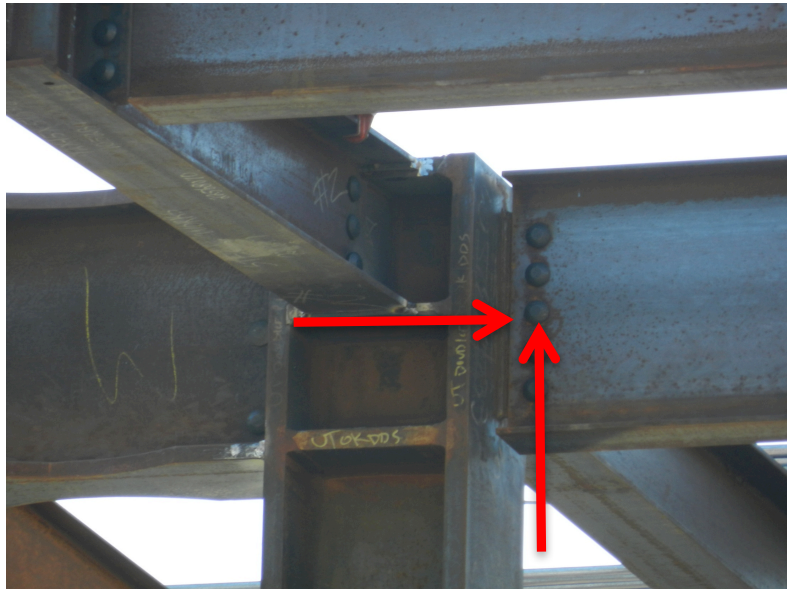
Reactive force with unknown magnitude and unknown direction

Support or Connection	Reaction	Number of Unknowns
 Frictionless pin or hinge  Rough surface	 Force of unknown direction	2

Usually the most convenient way to express the two unknowns

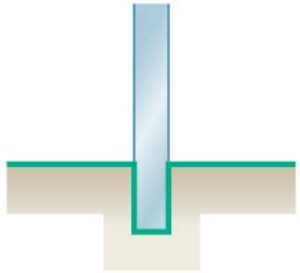
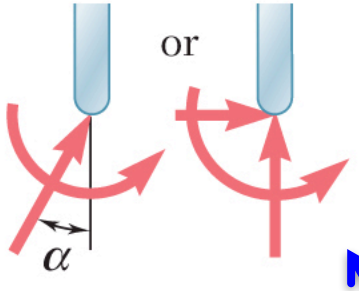
Examples pin supports and pin connections





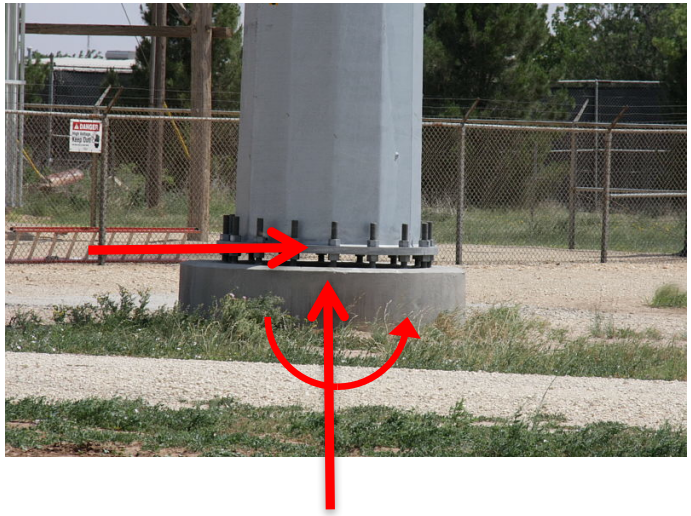
This beam connection where only the beam web is bolted to the column is usually modeled as a pin connection

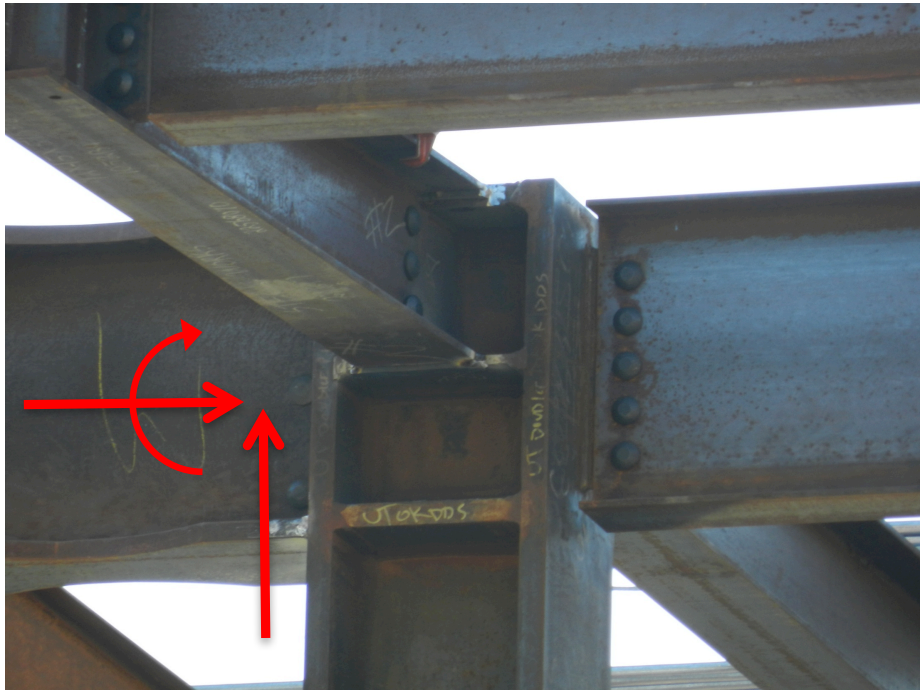
Reactive force with unknown magnitude, unknown direction, and an unknown moment

Support or Connection	Reaction	Number of Unknowns
 Fixed support	 Force and couple	3

Usually the most convenient way to express the three unknowns

Examples rigid supports and rigid connections





This beam connection where the beam flanges are welded to the column is usually modeled as a rigid (moment resisting) connection

Scalar Equations of Static equilibrium

General three-dimensional body

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

General two-dimensional (planar) body

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

Scalar Equations of Static equilibrium for concurrent force systems

Three-dimensional body with concurrent forces

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

Two-dimensional (planar) body with concurrent forces

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$