Centroid of an Area Using Integration

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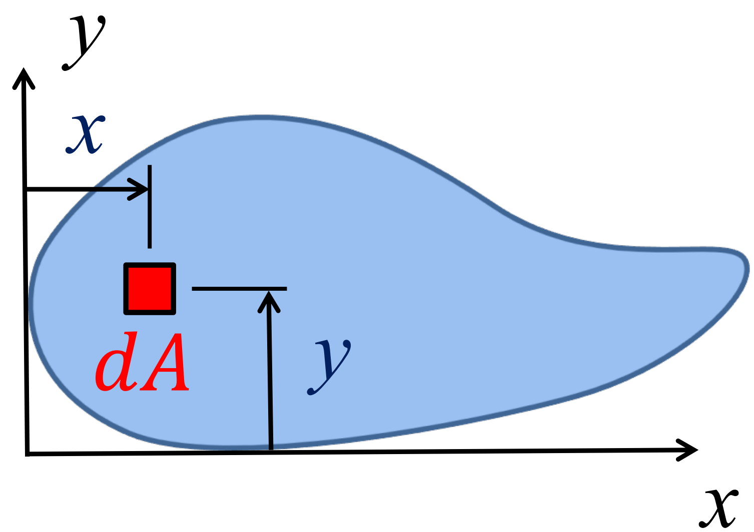
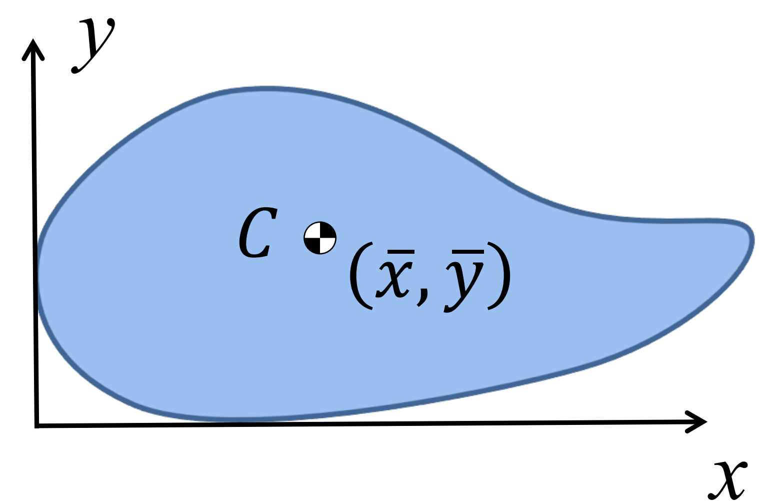
Centroid of an Area





$$A = \iint dA$$

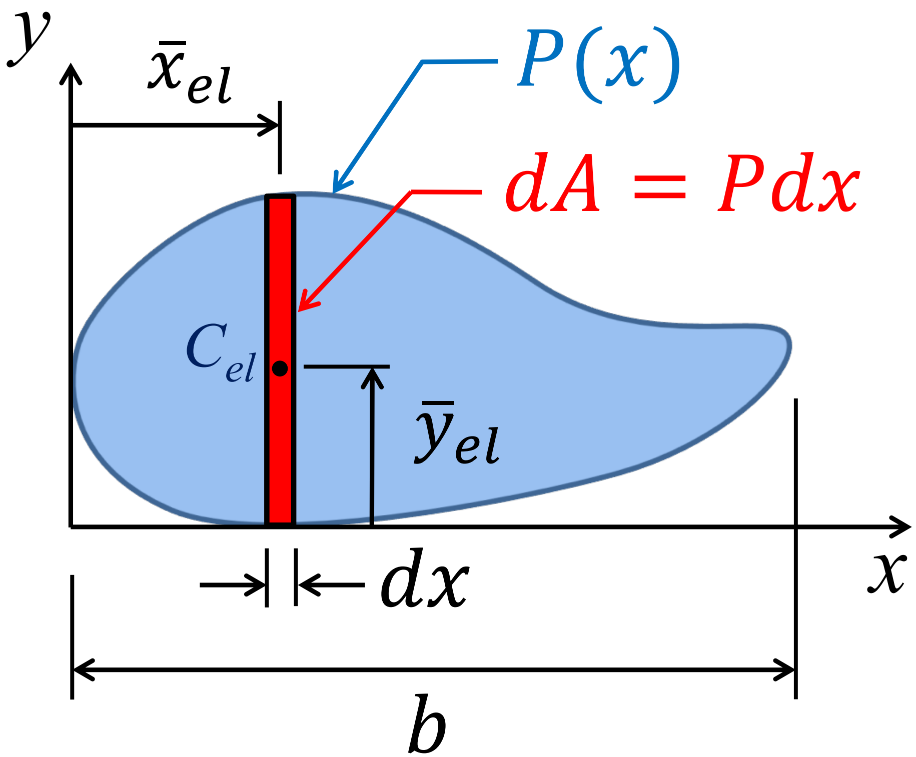
$$\bar{x} = \frac{\iint x dA}{A}$$

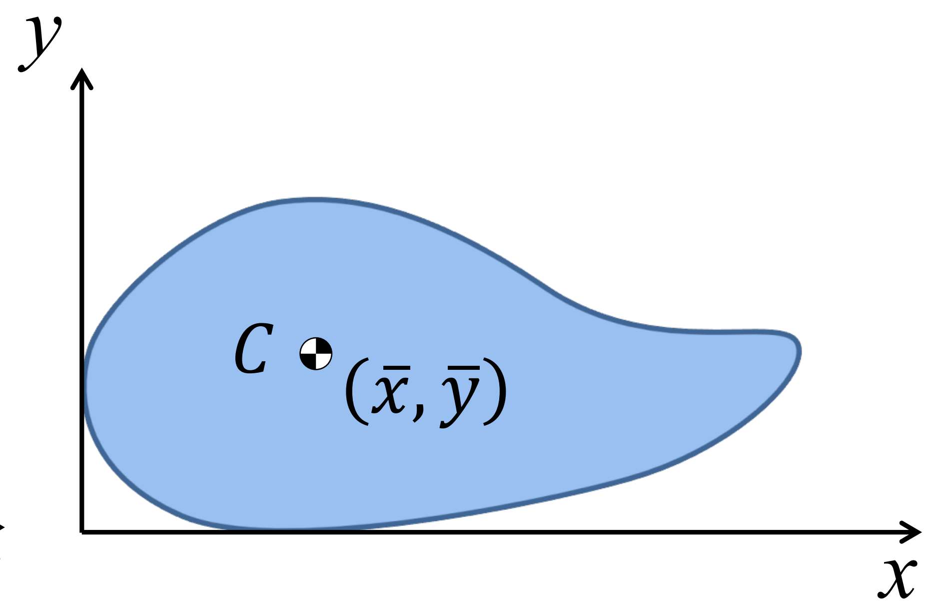
First moment of the area about the y axis

$$\bar{y} = \frac{\iint y dA}{A}$$

First moment of the area about the x axis

Divide the Area into Either Horizontal or Vertical Strips



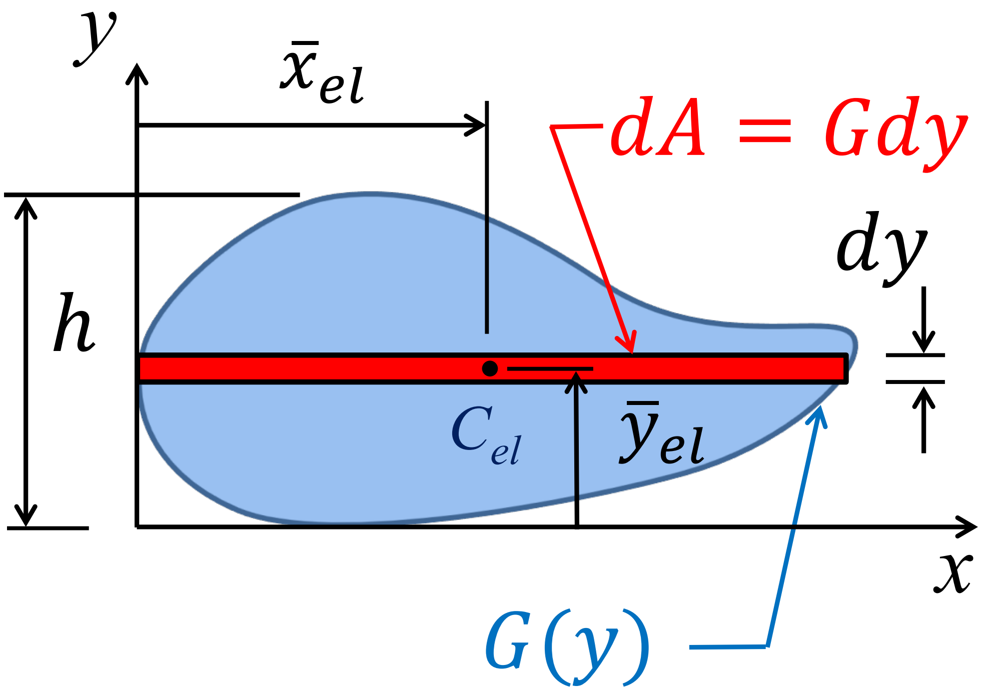


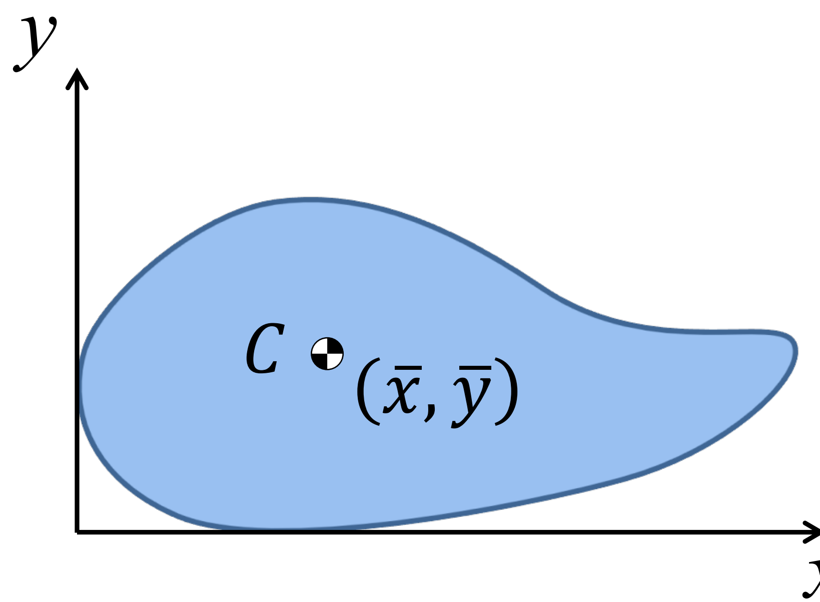
$$A = \int_0^b P dx$$

$$\bar{x} = \frac{\int_0^b \bar{x}_{el} \, P dx}{A}$$

$$\bar{y} = \frac{\int_0^b \bar{y}_{el} \, P dx}{A}$$

Divide the Area into Either Horizontal or Vertical Strips



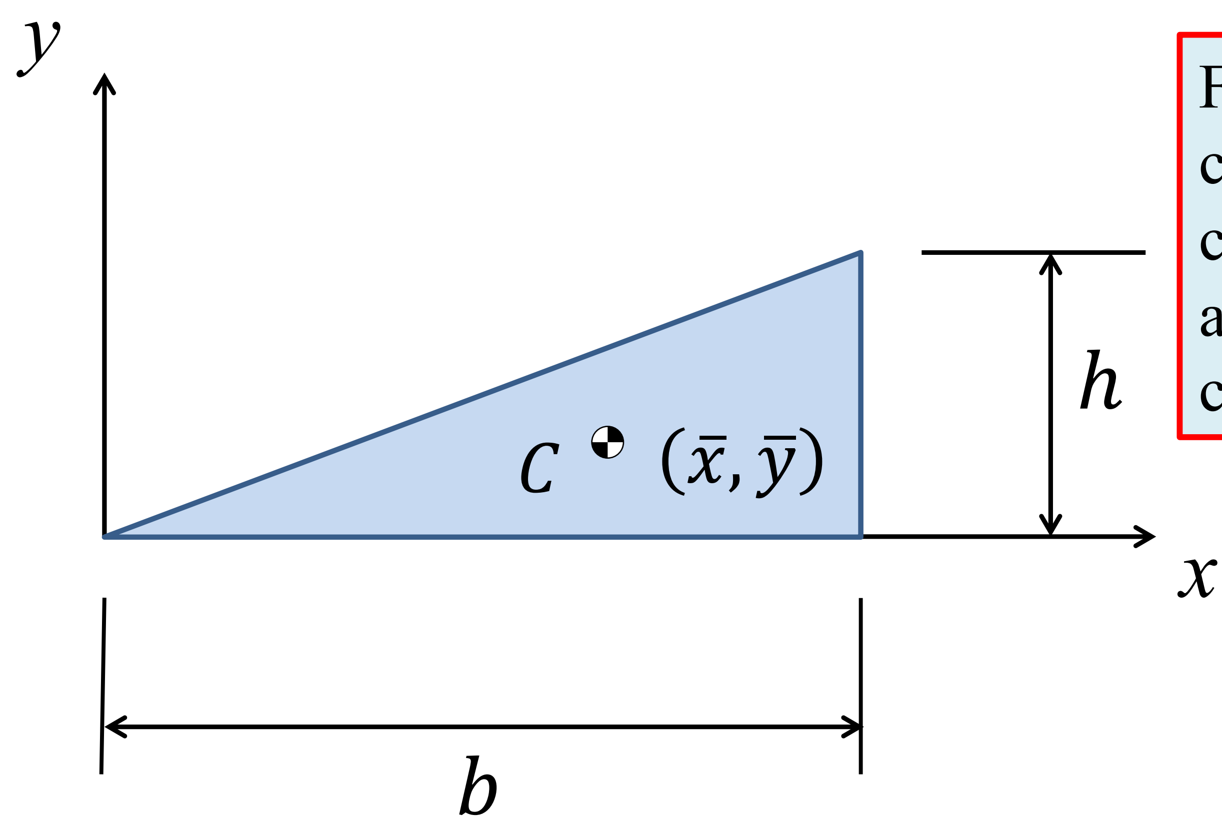


$$\bar{x} = \frac{\int_0^h \bar{x}_{el}\, Gdy}{A}$$

$$\bar{y} = \frac{\int_0^h \bar{y}_{el}\, Gdy}{A}$$

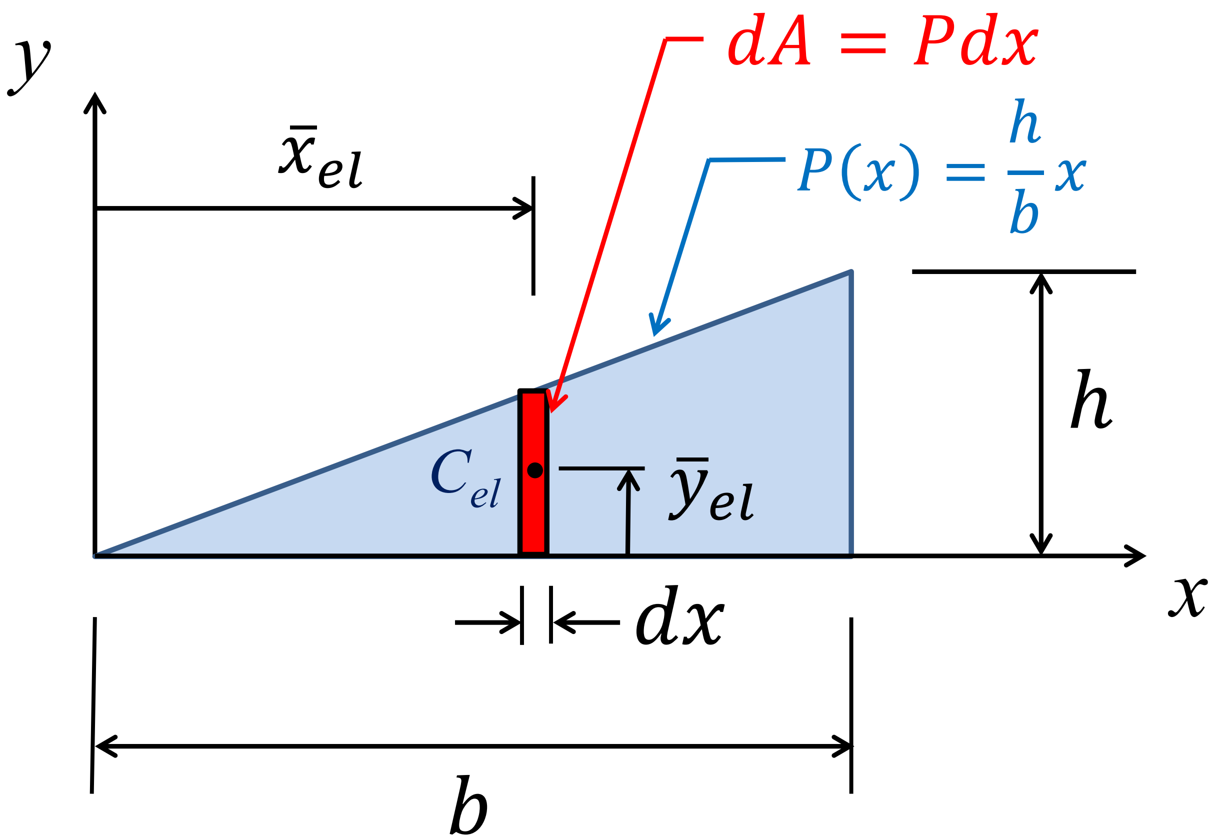
$$A = \int_0^h Gdy$$

Example Problem



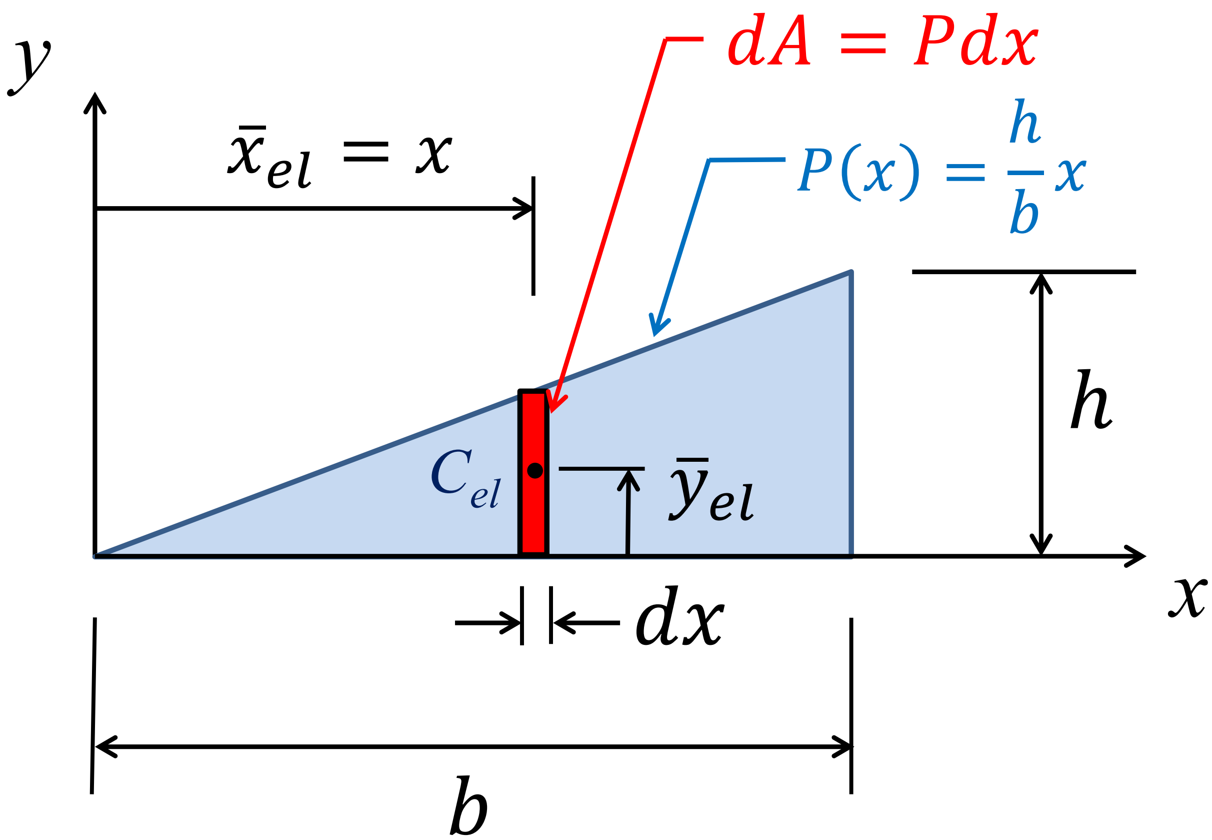
Find the *x* and *y* coordinates of the centroid of the shaded area with respect to the coordinate axes shown.

Divide Area into Vertical Strips



$$A = \int_0^b Pdx = \int_0^b \frac{h}{b} xdx = \frac{h}{b}\int_0^b xdx = \frac{h}{b}\left[\frac{x^2}{2}\right]_0^b = \frac{1}{2}bh$$

Find the *x* Coordinate of the Centroid

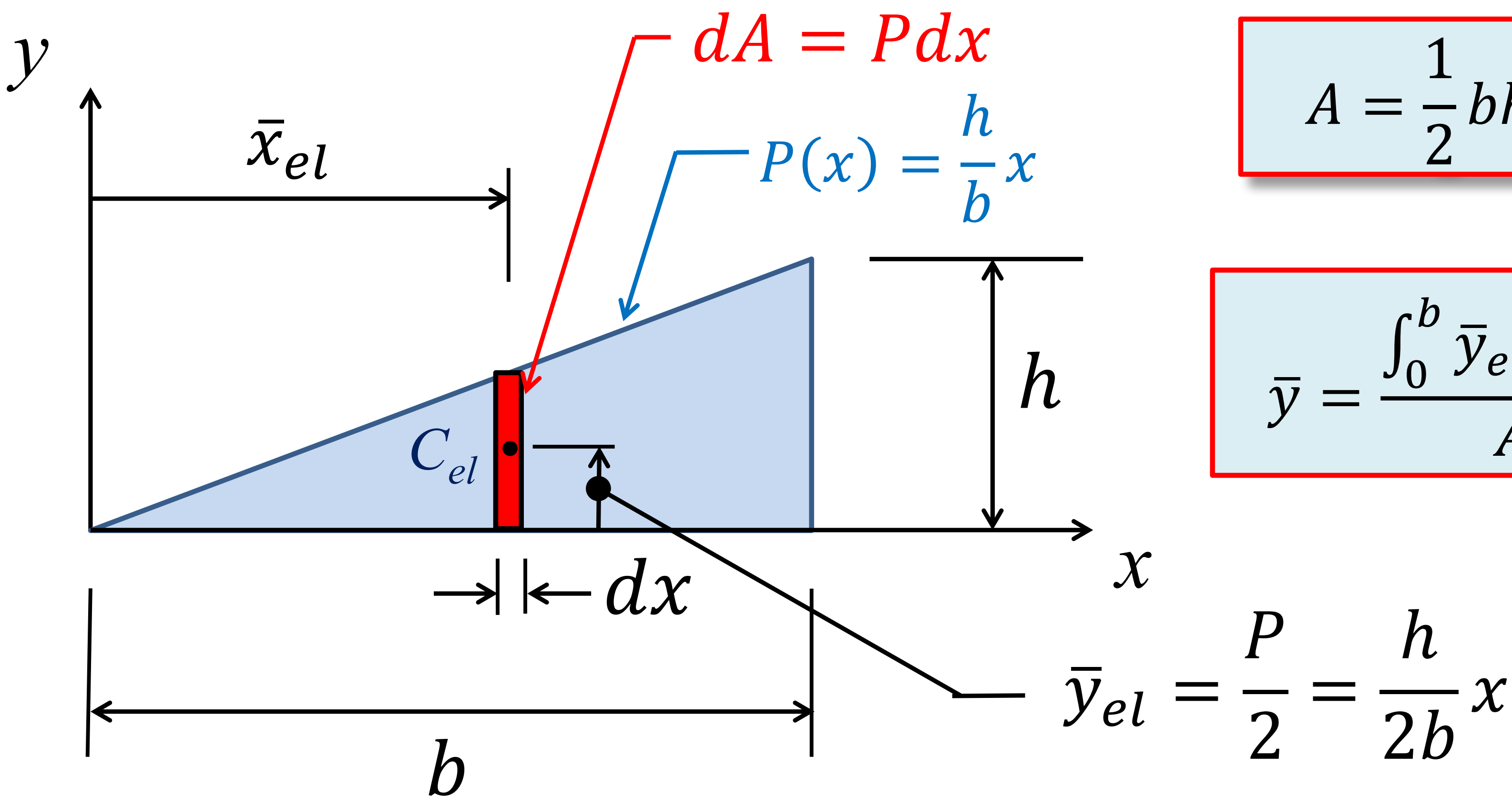


$$A = \frac{1}{2}bh$$

$$\bar{x} = \frac{\int_0^b \bar{x}_{el}\, P dx}{A}$$

$$\int_0^b \bar{x}_{el}\, P dx = \int_0^b x\left(\frac{h}{b}\right) x dx = \frac{h}{b}\int_0^b x^2\, dx = \frac{h}{b}\left[\frac{x^3}{3}\right]_0^b = \frac{1}{3}b^2 h$$

Find the y Coordinate of the Centroid

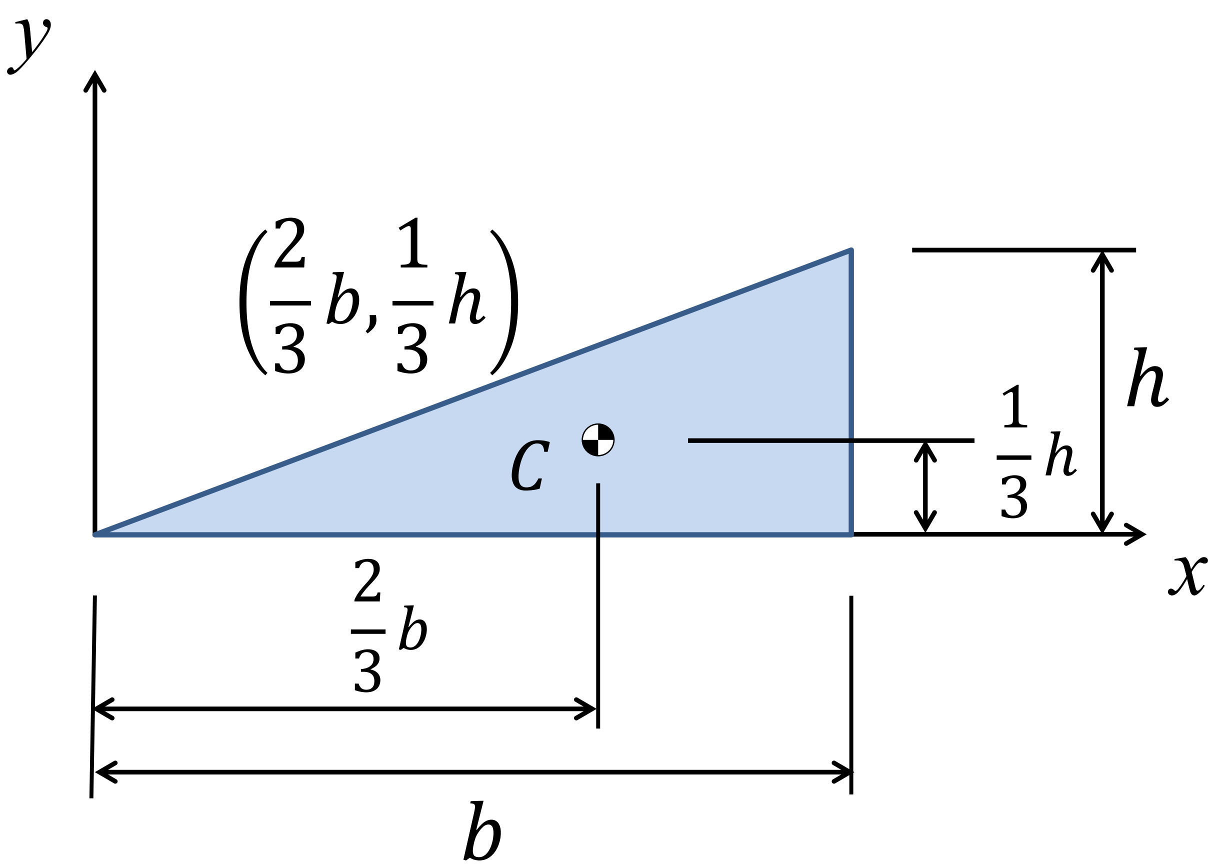


$$A = \frac{1}{2}bh$$

$$\bar{y} = \frac{\int_0^b \bar{y}_{el}\, P dx}{A}$$

$$\int_0^b \bar{y}_{el}\, P dx = \int_0^b \frac{h}{2b} x \left(\frac{h}{b}\right) x dx = \frac{h^2}{2b^2}\int_0^b x^2\, dx = \frac{h^2}{2b^2}\left[\frac{x^3}{3}\right]_0^b = \frac{1}{6}bh^2$$

Coordinates of the Centroid



$$A = \frac{1}{2}bh$$

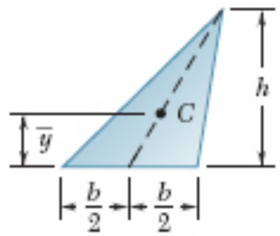
$$\bar{x} = \frac{\int_0^b \bar{x}_{el}\, P dx}{A}$$

$$\bar{y} = \frac{\int_0^b \bar{y}_{el}\, P dx}{A}$$

$$\bar{x} = \frac{\frac{1}{3}b^2h}{\frac{1}{2}bh} = \frac{2}{3}b$$

$$\bar{y} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} = \frac{1}{3}h$$

Result Agrees with the Tabulated Value for a General Triangular Area in Textbook

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$