Estimating the Publicness of Local Government Services: Alternative Congestion Function Specifications*

TOM S. MEANS
San Jose State University
San Jose, California

STEPHEN L. MEHAY
U. S. Naval Postgraduate School
Monterey, California

I. Introduction

The standard approach to estimating median voter expenditure equations has incorporated the specification of a congestion function, $\alpha(N)$, to convert the total output (Q) of a government provided good into individual consumption (q_i). In their classic papers, Borcherding and Deacon [2] and Bergstrom and Goodman [1] adopted a simple form, $\alpha(N) = N^{-\gamma}$. Using this form, estimated values of $\gamma = 0$ imply that the service in question is classified as a pure public good, while $\gamma = 1$ implies a private good. This approach is attractive from an empirical standpoint because it allows the data to determine the degree of publicness of the local government output. The empirical findings of Borcherding and Deacon, Bergstrom and Goodman, and numerous later studies that used the median voter approach,[1] indicate that most local government services do not exhibit a significant degree of publicness.[2]

Recent studies have questioned whether estimates of publicness are sensitive to the particular specification of the congestion relationship [6; 9]. Researchers also have objected to the standard form because it holds the degree of publicness fixed, which implies a decreasing marginal rate of congestion [3; 6; 15]. Moreover, holding the degree of publicness fixed implies a single estimated value of γ is sufficient to classify the publicness of the good. Critics of the standard approach argue that the degree of publicness should vary with population size. For instance, a local park may exhibit no congestion for small population sizes, but eventually become congested as population increases.

Unfortunately, no clear guidelines for specifying the congestion function are available. Edwards suggested that flexible functional forms are to be preferred because they "let the data speak for themselves" [6, 92]. He also argued that the preferred form is one that fits the data best. Based on these criteria, he concluded that a flexible form, such as an exponential form, is su-

*The authors thank Rodolfo Gonzalez for helpful comments.
1. See Gonzalez and Mehay [8] for a brief summary of this literature.
2. That is, the hypothesis $\gamma < 1$ is normally rejected.

perior to the standard specification in the median voter literature. When he used the exponential form, Edwards found that, in contrast to prior studies, three local government services displayed substantial publicness. Hayes and Slottje [9], on the other hand, investigated alternative specifications, including an exponential form, and concluded they were not superior on statistical grounds to the conventional approach.

Congestion functions that permit greater flexibility in measuring publicness offer clear advantages over the conventional form. Balanced against this, however, alternative and more complex congestion functions introduce several problems with respect to the estimation of publicness parameters in median voter demand functions. First, the alternative forms proposed in the recent literature do not always meet the simple criterion of "allowing the data to speak for themselves." Indeed, some of the proposed forms (in particular exponential-based forms) bias the estimated congestion parameters toward higher degrees of publicness (i.e., the amount of congestion is biased downward). Second, the alternative specifications of the congestion function involve more than one parameter to estimate. Gonzalez, Means, and Mehay [7] show that such specifications require joint tests on the parameters to test the pure public and private good hypotheses. Third, the proposed alternative forms yield non-nested models that require unnecessarily complicated procedures for testing whether an estimated equation provides a better fit of the data. Another related problem is that allowing the degree of publicness to vary with population size makes it difficult to apply conventional hypothesis testing methods. For the standard congestion function ($N^{-\gamma}$) the estimated value of γ can be tested as a point hypothesis (e.g., $\gamma = 0$ or $\gamma = 1$). When the degree of congestion varies with population size, the estimated degree of publicness must also vary. Some authors report the range (or frequency) of observations for which the good is less congested than a private good. This approach, however, cannot determine if these varying estimates of publicness are statistically different from a good that is purely private.[3] The clear advantage of holding the degree of congestion fixed is the ease in classifying the good as public, semi-public, or private. For the flexible functional forms, one cannot determine if a specific estimated value of the publicness parameter is different from unity.

One goal of this paper is to demonstrate the *potential* bias toward publicness contained in some of the alternative functional forms. Section II also demonstrates that the pure public and private good hypotheses can be tested as restricted expenditure equations without prior specification of the congestion function. Section III shows that the exponential- and population-based forms can be constructed as nested models and tested using conventional methods for determining whether alternative forms improve the explanatory power of the model. The simple nested hypothesis tests of the publicness parameters are developed in section IV and empirically estimated in section V.

II. A Median Voter Demand Model

In this section a reduced form expenditure equation is derived from a standard median voter model, in which the median voter's demand schedule is generated from a budget-constrained utility function. The flow of services to the local resident is defined by:

3. A similar problem occurs in demand estimation of consumer goods. A log-linear equation yields a single price elasticity, while a linear equation yields an infinite number of price elasticities. Hence, a log-linear model with an estimated elasticity greater than one always implies an elastic price response. Describing the response for a linear equation depends on the point chosen on the demand schedule. Depending on the variation of the observed prices, one could classify the responses as inelastic or elastic, making it difficult to compare the response to a price change between the linear and log-linear equations.

$$q_i = \alpha(N)Q \tag{1}$$

where q_i is individual i's consumption of the publicly provided output, Q. The budget constraint is written:

$$\begin{aligned} y_i &= Z + t_i(PQ/N) \\ &= Z + [t_i P/(\alpha(N)N)]q_i \\ &= Z + p_i q_i \end{aligned} \tag{2}$$

where y_i is individual income, Z is the amount spent on private goods, t_i is the individual's tax share (i.e., the taxpayer's share of the total budget) relative to per capita expenditures, P is the price of the output (Q), and p_i is the tax-price to the individual resident for q_i. Most median voter studies specify the budget constraint as $y_i = Z + t_i(PQ)$, where $0 \leq t_i \leq 1$. Following Dudley and Montmarquette [5], our tax share is defined relative to PQ/N, the per capita expenditure on the local public good.[4]

Assuming a multiplicative demand function,

$$q_i^d = A(p_i)^{\beta_1}(y_i)^{\beta_2}, \tag{3}$$

the log-linear expenditure equation is derived by specifying $E = PQ = P[q_i^d/\alpha(N)]$, substituting for q_i^d, and taking logs.[5]

$$\ln(E) = \ln(A) + \beta_1 \ln(t_i/N) - (1 + \beta_1)\ln(\alpha(N)) + \beta_2 \ln(y_i). \tag{4}$$

Expression (4) represents the standard reduced form equation used in the publicness literature. The derivation of (4) assumes that t_i may vary between voters but that the allocation of the good is the same for every voter.[6] An equal allocation of a pure public good requires $\alpha(N) = 1$ in order to set $q_i = Q$. For the pure private good case $\alpha(N) = 1/N$ in order to set $q_i = Q/N$. Substituting these restrictions into (4) yields one reduced form expenditure equation for the pure public good case:

$$\ln(E) = \ln(A) + \beta_1 \ln(t_i/N) + \beta_2 \ln(y_i), \tag{5}$$

and one for the private good case:

$$\ln(E) = \ln(A) + \beta_1 \ln(t_i) + \ln(N) + \beta_2 \ln(y_i). \tag{6}$$

The pure public and private good hypotheses can be tested by estimating (4) subject to the restrictions implied by (5) and (6). In terms of elasticities, the restriction for the pure public good

4. This has the advantage of showing the explicit relationship between the tax share and population. Moreover, any tax share definition can be rewritten in per capita terms if it is summed over N and $\Sigma t_i = 1$. Some studies define the tax share in terms of households (H). Substituting persons per household (N/H) still allows N to be extracted from the tax variable.

5. Most studies assume P is constant across municipalities and omit it from the final expenditure equation.

6. Denzau and Mackay [4] permit $\alpha(N)$ to vary between voters based on income. Our paper focuses on congestion functions that depend on N and yield equal allocation, as these represent the ones specified by authors who criticize the standard form of the congestion function for not allowing the degree of congestion to vary (and not because it forces equal allocation).

hypothesis is $\eta_{E,N} = -\eta_{E,t_i}(= -\beta_1)$. The restriction for the private good hypothesis is $\eta_{E,N} = 1$. Information from just these two restricted equations can determine how well each equation fits the data, whether the equations differ, and whether the restrictions on the coefficients imply a statistically significant change. Finally, the two restricted equations can be compared with the unrestricted equation that specifies $\alpha(N)$ in some form.

One advantage of testing the pure public and private good hypotheses as restricted equations is that an a priori specification of $\alpha(N)$ is not necessary. A second advantage is that, depending on the exact specification of $\alpha(N)$, the above method does not require the unraveling and testing of the parameter estimates from a nonlinear function. For example, the standard functional form $\alpha(N) = N^{-\gamma}$ yields $\eta_{E,N} = \gamma(1 + \beta_1) - \beta_1$. Testing $\gamma = 0$ or $\gamma = 1$ implies a nonlinear hypothesis test which requires extracting an estimated value for γ and its corresponding standard deviation. Borcherding and Deacon performed such a test by specifying the distribution of γ as a function of β_1 and $\eta_{E,N}$.[7] However, several other studies that used the same approach failed to perform the appropriate nonlinear hypothesis test. For example, Bergstrom and Goodman and MacMillan [12] failed to calculate the standard deviations necessary to test $\gamma = 0$,[8] and Edwards does not list the estimated standard deviations for his estimates of γ. The approach suggested in this paper avoids these estimation difficulties when testing the pure public and private good hypotheses.

III. Estimation Problems Using Alternative Congestion Functions

Several studies have proposed and tested alternative functional forms of the congestion function [3; 6; 9; 14]. We do not propose to test all of the many functional forms that have been suggested; however, the forms explored in this paper, while not exhaustive, represent a sample of the main forms that have appeared in the literature. The forms are listed in Table I: the first three are "population-based," the fourth represents an exponential function, and the fifth ("mixed") specification combines aspects of the previous two.[9]

We do not use the "increasing marginal congestion" function (IMC) that appears in Edwards because it permits *only* increasing marginal congestion (i.e., $\delta^2 q_i/\delta N^2 < 0$), given $\delta q_i/\delta N < 0$.[10] That is, even if the data suggest decreasing marginal congestion ($\delta^2 q_i/\delta N^2 > 0$), the IMC

7. Several median voter studies assume that the distributions for the estimates of β_1 and $\eta_{E,N}$ are independent. However, the correct specification for the estimated variance of γ will contain a covariance term for β_1 and $\eta_{E,N}$ [10, 541–48; 16, 244; 13, 101]. This covariance term will depend in part on $(X'X)^{-1}$ which will likely contain nonzero off-diagonal elements. Without knowing the sign of this covariance term, it is uncertain whether correctly incorporating the term would have a significant impact on the established results in the literature.

8. Failure to perform this test is probably due to the fact that most studies that employ the standard congestion function find that $\gamma > 1$, which indicates that $H_o: \gamma = 0$ would likely be rejected and $H_o: \gamma = 1$ would not be rejected. However, by failing to calculate the estimated standard deviation for γ, one does not know whether $H_o: \gamma = 0$ is rejected. As shown below (and in Gonzalez, Means, and Mehay [7]) there may be instances where neither hypothesis is rejected. That is, in some cases the data may not be able to distinguish between the pure public and private good hypotheses.

9. The first specification is the traditional form, which exhibits decreasing marginal congestion for $\gamma > 0$. The second population-based form was proposed by McKinney [14], while Hayes and Slottje [9] introduced the third population-based form. Edwards [6] used the exponential form; Hayes and Slottje, and McKinney used simpler exponential congestion functions that excluded the higher order terms. The final form, "mixed", is based on Buchanan's club theory [6].

10. We also do not use the IMC and Generalized Congestion functions in part because they cannot be tested as nested forms, and in part because neither performed well in Edwards' empirical tests. Craig [3] also uses an IMC form, but he uses it to measure congestion in the production of both an intermediate public output as well as a final output. Craig's focus is on service production and distribution *within* a city, and he does not use a median voter approach to

Table I. Alternative Congestion Function Specifications

Population-Based Congestion Functions	
I:	$\alpha(N) = N^{-\gamma}$
II:	$\alpha(N) = N^{-(\gamma+\alpha N)}$
III:	$\alpha(N) = N^{-(\gamma+\alpha \ln N)}$
Exponential Congestion Function	
IV:	$\alpha(N) = \exp\{aN + bN^2 + dN^3 - (a + b + d)\}$
"Mixed" Congestion Function	
V:	$\alpha(N) = N^{-\gamma}\exp\{a(1 - N)\}$

function forces the estimated expenditure equation toward increasing marginal congestion. This is an inherent flaw since testing the private good hypothesis ($\alpha(N) = 1/N$) requires that $\alpha(N)$ allow for the possibility that congestion decreases at the margin. Given the extensive prior empirical findings that have categorized local government goods as private in nature, it is inappropriate to specify a congestion function that does not allow decreasing marginal congestion as one possibility. All of the forms in Table I permit estimation that implies decreasing marginal congestion.

Although the exponential-based forms allow for decreasing marginal congestion, they are still biased toward exhibiting some degree of publicness when testing the private good hypothesis. For the private good hypothesis, $\alpha(N) = 1/N$, which implies $q_i = Q/N$, which in turn restricts the population elasticity (in equation (6)) to a value of unity. To see the bias, specify $\alpha(N) = \exp\{aN\}$ and try to set $q_i = Q\exp\{aN\}$ equal to Q/N. The solution requires:[11]

$$a = -\ln(N)/N. \tag{7}$$

Since the parameter on the left-hand-side of the expression is fixed, $q_i = Q/N$ only when the condition implied by expression (7) is met.

To demonstrate the potential estimation problem, Figure 1 graphs the relationship between population (N) and individual consumption (q_i). The horizontal line represents the pure public good line for an assumed output of 10 units, while the private good line is graphed as Q/N. The area between these two lines represents an area where q_i exhibits some degree of publicness. The exponential function $q_i = Q\exp\{aN\}$ is graphed by further assuming $a = -0.34657$. This particular congestion function forces q_i to cross the private good line at $N = 2$ and again at $N = 4$.

Assume that cities with different populations produce 10 units of a good that is private. The observations for q_i and N will all lie on the private good line. The estimated line for q_i using $\alpha(N) = \exp\{aN\}$ will be above Q/N for small cities (where $2 < N < 4$), which implies that the local government good exhibits some degree of publicness.[12] Similarly, for very large cities the

specify the estimating equations, making comparisons with the main literature difficult. But, regardless of the approach adopted, as we demonstrate below, the IMC function cannot perform well if local output is truly private.

11. For convenience we employ the simple exponential form used by Hayes and Slottje, which violates a property that Edwards and others would impose on congestion functions: if $N = 1$, then $q_i = Q$. This defect can easily be corrected by changing the expression to $q_i = Q\exp\{aN - a\}$, which is a restricted version of the exponential form. This adjustment meets the property that $q_i = Q$ when $N = 1$, but does not remove the bias of forcing the function to exhibit some degree of publicness as shown in Figure 1.

12. Assume the observations for q_i and N are distributed such that the estimated value of $a = -0.34657$ and that N is measured in units of 10,000.

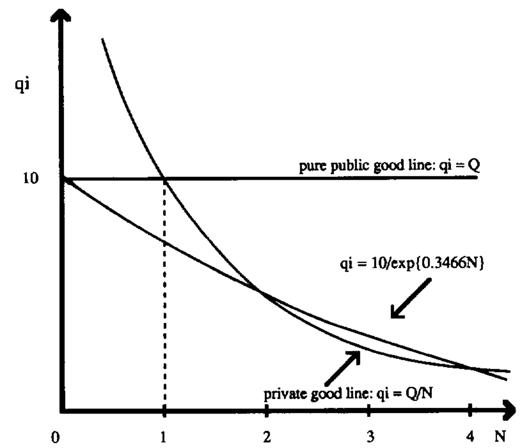

Figure 1

local service becomes more congested than a private good.[13] For the more general exponential form, the third-order polynomial is still biased and permits more potential crossovers with the private good line.

Although the exponential-based forms permit the degree of congestion to vary with N, they cannot be restricted to the case where the data requires the degree of congestion to be constant, as in the case of a private good.[14] The exponential forms *require* rather than *permit* the expenditure elasticity (with respect to population) to vary. Conversely, the population-based forms permit the degree of congestion to be estimated as varying or fixed depending on the distribution of the data. For these forms it is possible to set $q_i = Q/N$, when all observations lie on the private good line as described above. Also these forms can reduce to $N^{-\gamma}$, the traditional form. (To see this, consider form III where $\alpha(N) = N^{-(\gamma + \alpha \ln N)}$. Setting $\alpha(N) = 1/N$ requires that $\alpha = 0$ which reduces $\alpha(N)$ to $N^{-(\gamma)}$).[15]

An alternative way to show the estimation problem associated with the exponential congestion function is to compare the reduced form private good and public good expenditure equations. From expression (6) we know that the private good expenditure equation has a population elasticity of unity. If $\alpha(N) = \exp\{aN - a\}$ is substituted into (4), the expenditure equation becomes (in the equations that follow control variables are omitted to simplify the presentation):

$$\ln(E) = -\beta_1 \ln(N) - [a(1 + \beta_1)](N), \tag{8}$$

or expressed in terms of the population elasticity,

$$\eta_{E,N} = d \ln(E)/d \ln(N) = -\beta_1 - [a(1 + \beta_1)](N). \tag{9}$$

13. Both Edwards [6] and Craig [3] note that the alternative forms imply a non-constant rate of congestion. Edwards reports the percentage of observations where the predicted value of q_i is less than Q/N as an attempt to show the frequency of publicness. Craig reports the population sizes where the local public good is more congested than a private good. These statistics are not very meaningful if the underlying locally produced good is actually private as described above and the degree of congestion is estimated using an exponential-based function.

14. The exponential based forms do permit congestion to be fixed, but only in the case of a pure public good ($q_i = Q$).

15. Similar restrictions reduce both form II ($\alpha = 0$) and "mixed" ($a = 0$) to $N^{-\gamma}$.

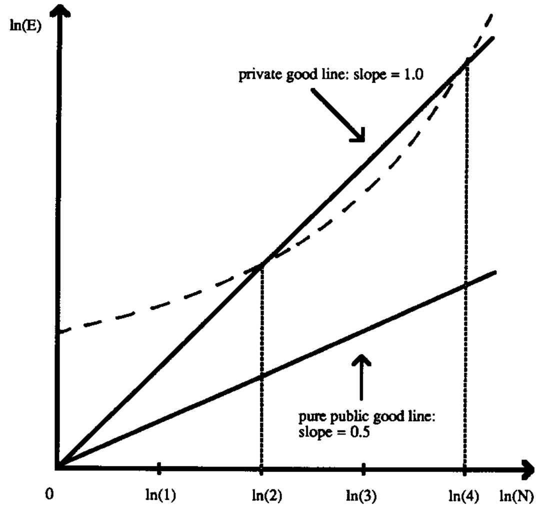

Figure 2

It should be clear that when $\beta_1 \neq -1$, it is not possible to have $\eta_{E,N} = 1$, which is required for the private good case.[16] The pure public good case requires $\eta_{E,N} = -\beta_1$ which is obtained by setting $a = 0$.

Figure 2 attempts to demonstrate these empirical relationships; it represents an extension of Figure 1, but graphed in $\ln(E) - \ln(N)$ space. Assume again that all the data points are generated so as to lie on the private good line (slope $= 1.0$ since the local service is actually private). The nonlinear dashed line represents the OLS estimates of expenditure equation (8). The pure public good line has a slope $= 0.5$.[17] As seen in Figure 2, the estimated equation will be forced to display some degree of publicness (i.e., toward the pure public good line) for smaller cities even though the observations have been generated by a private good.

IV. Nested Hypothesis Testing

By extracting the population variable from the tax share, the performance of the congestion functions can be tested by means of nested hypothesis tests. There are advantages to specifying the forms as nested versions of each other. First, because the nested forms are restricted versions of a more general model, the standard classical testing procedures can be employed to determine whether the nested model significantly reduces the explanatory power of the unrestricted model. If the unrestricted equation does not provide a significant increase in explanatory power, one should prefer the simpler nested version. Second, this approach avoids the disadvantages associated with non-nested models. Among other problems, the non-nested procedures require the specification of an artificial model, the test may not be able to identify the correct competing hypothesis, and the statistical tests are only asymptotically valid.[18]

16. In the case where $\beta_1 = -1$, it is impossible to identify the good as either pure public or private since the population elasticities for both reduced forms are the same (see equations (5) and (6)).

17. For the pure public good line the price elasticity is assumed to equal -0.50.

18. For a thorough presentation of non-nested models and the problems associated with the testing procedures, see MacKinnon [11].

Based on the five congestion functions, the median voter expenditure equations are specified as follows:[19]

$$\ln(E) = \ln(A) + \beta_1 \ln(t_i/N) + [(1+\beta_1)\gamma]\ln(N) + \beta_2 \ln(y_i)$$
$$= A_0 + A_1 \ln(N) + \beta_1 \ln(t_i) + \beta_2 \ln(y_i) \qquad \text{“Simple”} \quad (10)$$

$$\ln(E) = \ln(A) + \beta_1 \ln(t_i/N) + (1+\beta_1)(\gamma + \alpha N)\ln(N) + \beta_2 \ln(y_i)$$
$$= A_0 + A_1 \ln(N) + A_2(\ln(N) * N) + \beta_1 \ln(t_i) + \beta_2 \ln(y_i) \qquad \text{“Interaction”} \quad (11)$$

$$\ln(E) = \ln(A) + \beta_1 \ln(t_i/N) + (1+\beta_1)(\gamma + \alpha \ln(N))\ln(N) + \beta_2 \ln(y_i)$$
$$= A_0 + A_1 \ln(N) + A_3(\ln(N))^2 + \beta_1 \ln(t_i) + \beta_2 \ln(y_i) \qquad \text{“Quadratic”} \quad (12)$$

$$\ln(E) = \ln(A) + \beta_1 \ln(t_i/N) - (1+\beta_1)[aN + bN^2 + dN^3 - (a+b+d)] + \beta_2 \ln(y_i)$$
$$= A_0 + A_1 \ln(N) + B_1 N + B_2 N^2 + B_3 N^3 + \beta_1 \ln(t_i) + \beta_2 \ln(y_i) \qquad \text{“Exponential”} \quad (13)$$

$$\ln(E) = \ln(A) + \beta_1 \ln(t_i/N) + [\gamma(1+\beta_1)]\ln(N) - (1+\beta_1)a(1-N) + \beta_2 \ln(y_i)$$
$$= A_0 + A_1 \ln(N) + B_1 N + \beta_1 \ln(t_i) + \beta_2 \ln(y_i). \qquad \text{“Mixed”} \quad (14)$$

The “simple” form is a nested version of the other four forms. If $A_2 = A_3 = B_1 = B_2 = B_3 = 0$, then “simple” provides the best fit of the data and no explanatory power is gained by assuming an alternative form. If $A_1 \neq 0$ and $B_1 \neq 0$, but $A_2 = A_3 = B_2 = B_3 = 0$, then the “mixed” form is superior in terms of explaining the data.[20] To test the different functional forms, we estimate seven median voter demand equations for each of four municipal services. Two of the equations represent the restricted expenditure equations for testing the pure public and private good hypotheses: equations (5) and (6). The remaining five equations represent equations (10) through (14). For equation (13) two restrictions are tested: (a) $B_1 = B_2 = B_3 = 0$, and (b) $B_2 = B_3 = 0$. These two tests allow comparisons between (10) and (13), and between (13) and (14). For equations (11), (12) and (14), the t-statistics for the estimated values of A_2, A_3, and B_1 will determine whether there is any increase in explanatory power compared to (10).

V. Empirical Tests

Expenditure equations are estimated for four local government services: police protection, fire protection, parks and recreation, and sanitation. The control variables are median age, percent housing owner occupied, intergovernmental aid per capita, percent change in population (1970–1980), percent below poverty line, and percent nonwhite. The data are taken from two U.S. Census Bureau sources, the *1983 County and City Data Book* and the *1982 Census of Governments*, and represent California municipalities with populations between 25,000 and 250,000.

Tables II–V report the estimated OLS coefficients for the four expenditure categories. Each table reports estimates only for the tax share (t), median income (y), and population variables; to

19. These expenditure functions are derived from (4) with the appropriate substitution for $\alpha(N)$.

20. Note that not all of the equations are nested versions of each other. For example, “interaction” and “exponential” are not nested equations whereas “mixed” is a nested version of “exponential”. However, “simple” is a nested model for all of the alternative forms. One of the purposes of this paper is to determine whether there is any evidence to support a form other than the traditional form.

Table II. Police Protection (Sample Size = 132)

Variable	Public	Private	Forms				
			Simple	Interaction	Quadratic	Exponential	Mixed
Constant	−1.356	2.136[c]	1.958	2.180[c]	2.377[c]	2.153[c]	2.180[c]
	(−0.656)	(1.736)	(1.594)	(1.770)	(1.890)	(1.703)	(1.770)
$\ln(t_i)$	−0.639[a]	−0.024	−0.041	−0.030	−0.031	−0.027	−0.030
	(−9.202)	(−0.427)	(−0.736)	(−0.532)	(−0.542)	(−0.477)	(−0.531)
$\ln(y_i)$	0.876[a]	0.416[a]	0.408[b]	0.403[b]	0.402[b]	0.395[b]	0.402[b]
	(3.297)	(2.614)	(2.573)	(2.553)	(2.549)	(2.478)	(2.551)
$\ln(N)$	0.639[a]	1.000	1.078[a]	0.916[a]	0.653[b]	1.514	0.860[a]
	(9.202)	*	(21.748)	(7.690)	(2.177)	(1.278)	(5.576)
$\ln(N)*N$	—	—	—	0.007	—	—	—
				(1.493)			
$[\ln(N)]^2$	—	—	—	—	0.114	—	—
					(1.436)		
N	—	—	—	—	—	−0.242	0.030
						(−0.523)	(1.493)
N^2	—	—	—	—	—	0.016	—
						(0.613)	
N^3	—	—	—	—	—	0.0003	—
						(−0.624)	
R^2	0.5180	0.8330	0.8363	0.8393	0.8390	0.8398	0.8393
ADJ-R^2	0.4908	0.8236	0.8257	0.8274	0.8272	0.8251	0.8274
F-Stat.	19.039[a]	88.368[a]	78.561[a]	70.777[a]	70.664[a]	57.181[a]	70.777[a]
Test 1	9.049[a]	1.568	—	—	—	0.8639	—
Test 2	—	—	—	—	—	0.1966	—

Note: t-statistics in parentheses.
*Coefficient restricted to equal one.
a. Rejects null hypothesis ($\beta = 0$) at 1% level.
b. Rejects null hypothesis ($\beta = 0$) at 5% level.
c. Rejects null hypothesis ($\beta = 0$) at 10% level.

conserve space coefficients of the control variables are omitted. The first two columns of each table report the estimates of the restricted private and public good equations. Column three reports estimates of form "simple", which employs the traditional congestion specification. Columns 4–7 report the alternative four forms based on the expenditure equations specified in (11), (12), (13) and (14), respectively. For each table we report R^2 and adjusted R^2, along with the results of testing the restrictions on each equation. The first two columns, labeled "private" and "public" report a t-statistic test ("Test 1") of the significance of imposing the linear restriction (from (5) and (6)) on the parameters. A low t-statistic indicates that the change in the error sum of squares is insignificant from imposing the restriction. For the exponential form, F-statistics are reported for testing the nested versions, "simple" and "mixed".

The results for the estimated price and income elasticities are consistent for all spending categories. The estimated price elasticities for the tax variable are always negative and the estimated income elasticities are all positive. The restricted public good equation (column 1) always yields the highest price and income elasticities, while estimated elasticities for the private good

Table III. Fire Protection (Sample Size = 106)

Variable	Public	Private	Forms				
			Simple	Interaction	Quadratic	Exponential	Mixed
Constant	−3.585	0.239	0.090	0.139	0.169	0.065	0.135
	(−1.406)	(0.156)	(0.059)	(0.090)	(0.109)	(0.042)	(0.088)
$\ln(t_i)$	−0.687[a]	−0.018	−0.040	−0.036	−0.037	−0.038	−0.036
	(−8.508)	(−0.280)	(−0.590)	(−0.518)	(−0.535)	(−0.546)	(−0.520)
$\ln(y_i)$	0.841[a]	0.447[b]	0.437[b]	0.437[b]	0.438[b]	0.443[b]	0.437[b]
	(2.690)	(2.388)	(2.340)	(2.328)	(2.332)	(2.313)	(2.329)
$\ln(N)$	0.687[a]	1.000	1.071[a]	1.022[a]	0.968[a]	0.869	1.008[a]
	(8.508)	*	(19.331)	(7.381)	(2.800)	(0.644)	(5.658)
$\ln(N) * N$	—	—	—	0.002	—	—	—
				(0.386)			
$[\ln(N)]^2$	—	—	—	—	0.027	—	—
					(0.300)		
N	—	—	—	—	—	0.090	0.008
						(0.173)	(0.371)
N^2	—	—	—	—	—	−0.006	—
						(−0.212)	
N^3	—	—	—	—	—	0.0002	—
						(0.255)	
R^2	0.5169	.8307	0.8335	0.8338	0.8337	0.8341	0.8338
ADJ-R^2	0.4824	.8186	0.8198	0.8182	0.8181	0.8147	0.8182
F-Stat.	14.977[a]	68.700[a]	60.713[a]	53.510[a]	53.471[a]	42.968[a]	53.502[a]
Test 1	8.015[a]	1.278	—	—	—	0.1094	—
Test 2	—	—	—	—	—	0.0966	—

Note: t-statistics in parentheses.
*Coefficient restricted to equal one.
a. Rejects null hypothesis ($\beta = 0$) at 1% level.
b. Rejects null hypothesis ($\beta = 0$) at 5% level.
c. Rejects null hypothesis ($\beta = 0$) at 10% level.

equation are closer to those from the unrestricted equations. The income coefficient is statistically significant in all estimates, whereas the tax share coefficient is statistically significant only in column one.

For the three main services—police, fire, and parks and recreation—the results in Tables II–IV are consistent across services and strongly support the conclusion that these services can be characterized as private goods. The private good equation always outperforms the public good equation. The t-statistics (see "Test 1") are always insignificant for restricting the population elasticity to one, whereas the t-statistics for restricting the elasticity to minus the tax share elasticity are significant. The R^2's for the private good equations are approximately 35 to 60 percent higher than for the pure public good equations.

The two restricted equations can also be compared to the unrestricted equations using an F-test. From Table II the unrestricted R^2 of the "exponential" equation equals 0.8398 whereas the restricted "public" and "private" restricted equations yield R^2's of 0.5180, and 0.8330, respectively. The critical F-value (4, 119) is approximately 3.48 (1% level), which implies a cutoff value

Table IV. Parks and Recreation (Sample Size = 136)

Variable	Public	Private	Forms: Simple	Interaction	Quadratic	Exponential	Mixed
Constant	−8.693[a]	−5.074[b]	−5.284[b]	−5.093[b]	−4.987[b]	−5.362[b]	−5.100[b]
	(−3.468)	(−2.411)	(−2.518)	(−2.406)	(−2.301)	(−2.464)	(−2.409)
$\ln(t_i)$	−0.729[a]	−0.154	−0.185[c]	−0.174[c]	−0.177[c]	−0.182[c]	−0.175[c]
	(−8.263)	(−1.577)	(−1.856)	(−1.736)	(−1.762)	(−1.798)	(−1.740)
$\ln(y_i)$	1.405[a]	0.975[a]	0.954[a]	0.949[a]	0.949[a]	0.972[a]	0.949[a]
	(4.256)	(3.513)	(3.449)	(3.425)	(3.423)	(3.485)	(3.424)
$\ln(N)$	0.729[a]	1.000	1.135[a]	0.986[a]	0.827	0.292	0.943[a]
	(8.263)	*	(12.835)	(4.570)	(1.518)	(0.136)	(3.375)
$\ln(N) * N$	—	—	—	0.007	—	—	—
				(0.758)			
$[\ln(N)]^2$	—	—	—	—	0.114	—	—
					(1.436)		
N	—	—	—	—	—	0.387	0.027
						(0.462)	(0.724)
N^2	—	—	—	—	—	−0.026	—
						(−0.560)	
N^3	—	—	—	—	—	0.0006	—
						(0.665)	
R^2	0.4871	0.6519	0.6582	0.6597	0.6591	0.6627	0.6596
ADJ-R^2	0.4590	0.6329	0.6366	0.6354	0.6347	0.6328	0.6353
F-Stat.	17.362[a]	34.243[a]	30.567[a]	27.143[a]	27.063[a]	22.151[a]	27.127[a]
Test 1	6.535[a]	1.520	—	—	—	0.5585	—
Test 2	—	—	—	—	—	0.5775	—

Note: t-statistics in parentheses.
*Coefficient restricted to equal one.
a. Rejects null hypothesis ($\beta = 0$) at 1% level.
b. Rejects null hypothesis ($\beta = 0$) at 5% level.
c. Rejects null hypothesis ($\beta = 0$) at 10% level.

for the restricted R^2 of 0.8222. In this case the "private" restricted equation does not result in a significant reduction in explanatory power, whereas the opposite is true for the "public" restricted equation.[21]

Turning to the estimates for sanitation, the results are less straightforward. Overall, the seven estimated equations are similar in explanatory power. The overall F-values for sanitation are smaller when compared to the three other services, but are still statistically significant at the one percent level. The results suggest that the data cannot distinguish, in a statistical sense, between the restricted public good equation and the private good equation. We conclude that sanitation could be labeled as either a pure public or private good: the data do not reveal a preference in terms of the degree of publicness of this service.

In the tests of the congestion functions, the traditional function (equation (10)) outperforms

21. To conserve space the remaining F-statistics comparing the restricted equations and other unrestricted equations are not presented. The other unrestricted equations have slightly lower R^2s, which has little effect on the cutoff value for the restricted R^2. Adjusting for the change in the degrees of freedom does not alter the basic conclusion: the "public" restricted equation is rejected, whereas the "private" restricted equation is not.

Table V. Sanitation (Sample Size = 105)

			Forms				
Variable	Public	Private	Simple	Interaction	Quadratic	Exponential	Mixed
Constant	−6.779	−5.252	−4.911	−4.574	−4.346	−4.563	−4.523
	(1.620)	(1.258)	(1.172)	(1.079)	(1.005)	(1.046)	(1.078)
$\ln(t_i)$	−0.704[a]	−0.355	−0.302	−0.288	−0.292	−0.270	−0.288
	(4.847)	(1.579)	(1.308)	(1.233)	(1.252)	(1.133)	(1.235)
$\ln(y_i)$	1.233[b]	1.039[c]	1.087[b]	1.086[b]	1.086[b]	1.054[c]	1.085[b]
	(2.277)	(1.949)	(2.030)	(2.021)	(2.021)	(1.931)	(2.021)
$\ln(N)$	0.704[a]	1.000	0.847[a]	0.637[c]	0.319	2.366	0.567
	(4.847)	*	(5.408)	(1.681)	(0.337)	(0.616)	(1.464)
$\ln(N) * N$	—	—	—	0.008	—	—	—
				(0.608)			
$[\ln(N)]^2$	—	—	—	—	0.139	—	—
					(0.564)		
N	—	—	—	—	—	−0.691	0.037
						(0.467)	(0.805)
N^2	—	—	—	—	—	0.0417	—
						(0.506)	
N^3	—	—	—	—	—	0.0009	—
						(0.511)	
R^2	.3639	.3883	.3943	.3967	.3963	.3983	.3967
ADJ-R^2	.3180	.3442	.3439	.3395	.3352	.3272	.3397
F-Stat.	7.926[a]	8.797[a]	7.813[a]	6.940[a]	6.931[a]	5.598[a]	6.940[a]
Test 1	2.155[b]	0.977	—	—	—	0.206	—
Test 2	—	—	—	—	—	0.130	—

Note: t-statistics in parentheses.
*Coefficient restricted to equal one.
a. Rejects null hypothesis ($\beta = 0$) at 1% level.
b. Rejects null hypothesis ($\beta = 0$) at 5% level.
c. Rejects null hypothesis ($\beta = 0$) at 10% level.

the more flexible functional forms for all four service categories, including sanitation. In each category there is no statistically significant increase in explanatory power from using the more flexible forms. In fact, for fire protection and parks and recreation, the traditional form has the highest adjusted R^2. The data suggest that there is no support for using the more flexible functional forms to measure congestion. In this regard the results support Hayes and Slottje, who also found the conventional approach to be superior to alternative specifications.

The empirical results also clearly show that the alternative forms force the data to exhibit some degree of publicness. This point is demonstrated in Figure 3 in which the estimates from Table II are used to plot the actual relationships that were illustrated in Figure 2. In particular, Figure 3 plots the estimated lines for the private good equation (slope = 1.0), the equation using the traditional congestion function (the "simple" line), and the equation using the "mixed" form. As the plot shows, the estimated line for the "mixed" form predicts more publicness by dropping below both the "simple" line and the private good line at higher values of $\ln(N)$.[22]

22. Eventually the "mixed" line crosses back over the "private" line similar to Figure 2.

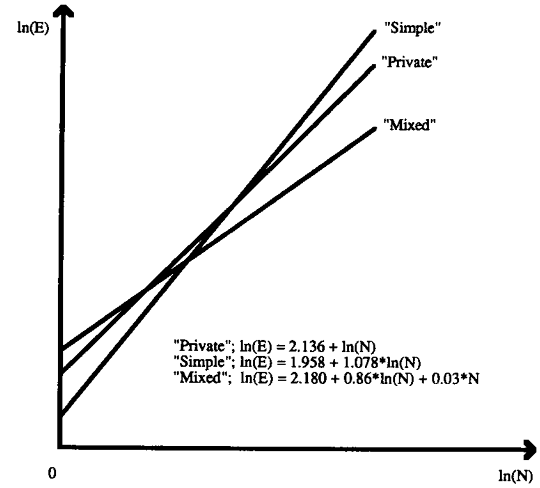

Figure 3

VI. Summary and Conclusion

The tests of the pure public good and private good hypotheses proposed in this paper have two principal advantages. First, by specifying the tax share in per capita terms, the population variable was extracted from the tax share, which allowed the exponential and population-based forms to be specified and tested as nested hypotheses. Second, we have shown that a priori specification of the form of $\alpha(N)$ is unimportant in estimating the restricted pure public and private good expenditure equations.

The conclusion appears warranted that attempts to use more flexible forms of the congestion function $\alpha(N)$ do not appear to provide promising avenues of future research. For every local service category, the results in this paper indicate that no significant increase in explanatory power is obtained by using the more flexible forms. Furthermore, flexible forms, as seen in the case of the exponential function, bias the data toward exhibiting some degree of publicness. Given the widespread empirical findings in the literature—that most local government services do not exhibit a significant degree of publicness—there would seem to be little justification for specifying forms that a priori impose some degree of publicness.

References

1. Bergstrom, Theodore C. and Robert P. Goodman, "Private Demands for Public Goods." *American Economic Review*, June 1973, 280–96.

2. Borcherding, Thomas E. and Robert T. Deacon, "The Demand for the Services of Non-federal Governments." *American Economic Review*, December 1972, 891–901.

3. Craig, Steven G., "The Impact of Congestion on Local Public Good Production." *Journal of Public Economics*, April 1987, 331–53.

4. Denzau, Arthur T. and Robert J. Mackay, "Benefit Shares and Majority Voting." *American Economic Review*, March 1976, 69–76.

5. Dudley, Leonard and Claude Montmarquette, "The Effects of Non-Clearing Labor Markets on the Demand for Public Spending." *Economic Inquiry*, April 1984, 151–70.

6. Edwards, J. H. Y., "Congestion Function Specification and the Publicness of Local Goods." *Journal of Urban Economics*, January 1990, 80–96.

7. Gonzalez, Rodolfo A., T. S. Means and Stephen L. Mehay, "Empirical Tests of the Samuelsonian Publicness Parameter: Has the Right Hypothesis Been Tested?" *Public Choice*, November 1993, 523–34.

8. Gonzalez, Rodolfo A. and Stephen L. Mehay, "Economies of City Size in a Price-Searcher Model of Local Government." *Public Finance*, 1987, 236–49.

9. Hayes, Kathy J. and Daniel J. Slottje, "Measures of Publicness Based on Demographic Scaling." *Review of Economics and Statistics*, November 1987, 713–18.

10. Judge, George G. et al. *Introduction to the Theory and Practice of Econometrics*, 2nd ed. New York: John Wiley & Sons, 1988.

11. MacKinnon, James G. "Model Specification Tests Against Non-Nested Alternatives." *Econometric Reviews*, 1983, 85–110.

12. MacMillan, Melville L. "On Measuring Congestion of Local Public Goods." *Journal of Urban Economics*, September 1989, 131–37.

13. Madala, G. S. *Econometrics*, New York: McGraw-Hill, 1977.

14. McKinney, Scott, "Crowding and the Club Membership Margin." *Journal of Urban Economics*, November 1987, 312–23.

15. Oates, Wallace E. "On the Measurement of Congestion in the Provision of Local Public Goods." *Journal of Urban Economics*, July 1988, 85–94.

16. Pindyck, Robert S. and Daniel L. Rubinfeld, *Econometric Models and Economic Forecasts*, 3rd ed. New York: McGraw-Hill, 1991.